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Modou NGOM and Tchilabalo A. KPANZOU

Weak Convergence (IA)

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Dedicatory.

To my wife Mbaye Ndaw Fall who is accompanying for decades with love and patience

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Modou Ngom participated in the project of the book while preparing his PhD thesis : **On a stochastic process doubly indexed, with margins estimating the extreme value index, and its Gaussian and non Gaussian Asymptotic**. He collaborated with Prof Gane Samb LO update this part of his PhD course in Weak convergence to produce the book.

Modou Ngom has also been a lecturer many years in Mathematics, Probability and Measure, at the University of Gaston Berger of Saint-Louis, Senegal, Saint-Louis, Senegal (UGB)

Dedicatory of Modou NGOM.

To my family, whose support, encouragement and faithful prayers arouse my envy to go forward.

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Weak Convergence (IA). Sequences of Random Vectors

ABSTRACT. (English) This monograph aims at presenting the core weak convergence theory for sequences of random vectors with values in $\mathbb{R}^k$. In some places, a more general formulation in metric spaces is provided. It lays out the necessary foundation that paves the way to applications in particular sub-fields of the theory. In particular, the needs of Asymptotic Statistics are addressed. A whole chapter is devoted to weak convergence in $\mathbb{R}$ where specific tools, for example for handling weak convergence of sequences using independent and identically distributed random variables such that the Renyi's representations by means of standard uniform or exponential random variables, are stated. The functional empirical process is presented as a powerful tool for solving a considerable number of asymptotic problems in Statistics. The text is written in a self-contained approach with the proofs of all used results at the exception of the general Skorohod-Wichura Theorem. We finish the book with a chapter on weak convergence of bounded measures and locally bounded measures in preparation of a more more general theory of measures on topological spaces

(Français) Cet ouvrage a l'ambition de présenter le noyau dur de la théorie de la convergence vague de suite de vecteurs aléatoires dans $\mathbb{R}^k$. Autant que possible, dans certaines situations, la théorie générale dans des espaces métriques est donnée. Il prépare la voie à une spécialisation dans certains sous-domaines de la convergence vague. En particulier, les besoins de la statistique asymptotique ont été satisfaits. Un chapitre de l'ouvrage concerne la convergence vague dans $\mathbb{R}$ avec des outils spécifiques, par exemple, pour étudier les suites de variables aléatoires indépendantes et identiquement distribuées tels que la représentation de Renyi au moyen de variables aléatoires uniformes ou exponentielles standard. Le processus empirique fonctionnel est introduit comme un outil puissant pour étudier des problèmes asymptotiques en Statistiques. L'ouvrage est rédigé dans une approche auto-citante avec toutes les preuves des résultats utilisés, à l'exception du Théorème de Skorohod-Wichura. Nous clôturons le livre par un chapitre sur la convergence vague de mesures bornées et de mesures localement bornées en prélude à une th théorie plus générale de convergence vague de mesures sur des espaces topologiques.

Keywords. Weak convergence; Convergence in distribution; Portmanteau Theorem; Probability Laws characterization; Distribution functions; Characteristic functions; Probability density functions; Random Walks; Empirical processes; Multinomial Laws; Relative compactness; Asymptotic and uniform tightness; Continuous mapping theorem; Rényi and Malmquist representations; Order Statistics; Multivariate Delta methods; Functional empirical process, weak convergence of bounded measure; Helly-Bray theorem.

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