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**A Course on Elementary Probability
Theory**

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General acknowledgment.

As main author, I wish to acknowledge the help of many people. I have been working for almost thirty years, mainly at the Saint-Louis Gaston University, since 1991. I also taught in many African Universities, especially at the master degree level. I worked in the administration of the university at all levels (head of department, dean faculty, vice-president). I also supervised more than twenty Ph.D. theses and several master dissertations. As well, I animated the LERSTAD, a research group I created around 1992 and ran it for years with a regular weekly seminar.

The books I am writing are the outcomes of all these activities. I worked with amazing people, younger colleagues and Ph.D. students who became later high profiled researchers in Africa, Europa, Canada and the United States, and other parts of the world. For the last few years, I have been teaching in Nigeria, in the African University of Sciences and Technology of the Nelson Mandela Institute, Abuja, Nigeria.

The books of this series in Mathematics in general, and in Random Analysis (Probability Theory and Statistics and their applications) are written in English since we want to reach a bigger public. But French versions will be published for books regularly used in undergraduate education. A broader presentation of our series of book can be found in the general preface, in page [1](#).

I want to thank many people involved in the publication process of our books, particularly this one:

(1) Students who followed this course at Université Gaston Berger and at Université Dakar-Bourguiba (SENEGAL) during years. The course I taught them has become the book you have in your hand.

(2) Members of my research teams (LERSTAD, IMHOTEP) who are asked to read all our books and who regularly take part in the editing (Drs. Tchilabola A. Kpanzou [Togo]; Harouna Sangaré, Soumaila Dembél'é, Mouminou Diallo [Mali]; Modou Ngom, Diam Bâ, Amadou Dadhié Ba; Mrs Gorgui Gning, Cherif Mamadou Moctar Traoré, etc.)

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The author acknowledges continuous and various support from the authorities of Gaston Berger University. It is true that in Senegalese public universities, the weekly teaching charge for full professors is five hours per week. The remaining time is devoted for supervision and research activities. Accordingly, scholars have all the means to realize their research activities : personal office, equipment with computers, printers, fax machines, ink, paper, internet connection, etc., and funding for participation in conferences.

In that sense, I acknowledge that writing the books of this series is implicitly funded by the university and the state of Senegal. I express my most sincere appreciations to their authorities.

Abstract of the book.

This book introduces to the theory of probabilities from the beginning. Assuming that the reader possesses the normal mathematical level acquired at the end of the secondary school, we aim to equip him with a solid basis in probability theory. The theory is preceded by a general chapter on counting methods. Then, the theory of probabilities is presented in a discrete framework. Two objectives are sought. The first is to give the reader the ability to solve a large number of problems related to probability theory, including application problems in a variety of disciplines. The second is to prepare the reader before he takes course on the mathematical foundations of probability theory. In this later book, the reader will concentrate more on mathematical concepts, while in the present text, experimental frameworks are mostly found. If both objectives are met, the reader will have already acquired a definitive experience in problem-solving ability with the tools of probability theory and at the same time he is ready to move on to a theoretical course on probability theory based on the theory of Measure and Integration. The book ends with a chapter that allows the reader to begin an intermediate course in mathematical statistics.

Keywords. combinatorics; discrete counting; elementary probability; equiprobability; events and operation on events; independence of events; conditional probabilities; Bayes' rules; random variables; discrete and continuous random variables; bi-dimensional random variable; probability laws; probability density functions; cumulative distribution functions; Independence of random variables; usual probability laws and their parameters; introduction to statistical mathematics; convex functions; .

AMS 2010 Classification Subjects : 60GXX; 62GXX.

Résumé de l'ouvrage.

Cet ouvrage introduit à la théorie des probabilités depuis le début. En supposant que le lecteur possède le niveau mathématique normal acquis à la fin du lycée, nous ambitionnons de le doter d'une base solide en théorie des probabilités. L'exposé de la théorie est précédé d'un chapitre général sur les méthodes de comptage. Ensuite, la théorie des probabilités est présentée dans un cadre discret. Deux objectifs sont recherchés. Le premier est de donner au lecteur la capacité à résoudre un grand nombre de problèmes liés à la théorie des probabilités, y compris les problèmes d'application dans une variété de disciplines. Le second était de préparer le lecteur avant qu'il n'aborde l'ouvrage sur les fondements mathématiques de la théorie des probabilités. Dans ce dernier ouvrage, le lecteur se concentrera davantage sur des concepts mathématiques tandis que dans le présent texte, il se trouvent surtout des cadres expérimentaux. Si les deux objectifs sont atteints, le lecteur aura déjà acquis une expérience définitive en capacité de résolution de problèmes de la vie réelle avec les outils de la théorie des probabilités et en même temps, il est prêt à passer à un cours théorique sur les probabilités basées sur la théorie de la mesure et l'intégration. Le livre se termine par un chapitre qui permet au lecteur de commencer un cours intermédiaire en statistiques mathématiques.

x

Dedication.

To our beloved and late sister Khady Kane LO
27/07/1953 - 7/11/1988



FIGURE 1. The ever smiling young lady

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