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Stock Price Prediction using Multidimensional Geometric Brownian Motion and DCC-GARCH Models

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Abstract. The primary focus of financial modeling is to understand how stock prices fluctuate over time. In this paper, we use two financial models, the Multidimensional Geometric Brownian Motion model and the DCC-GARCH model to analyze these movements. We use daily price data from three banks listed on the Johannesburg Stock Exchange to estimate the parameters of each model and test how well they can forecast returns and prices. Subsequently the models are assessed for their effectiveness in simulating returns and predicting price fluctuations using standard evaluation metrics.

Key words: stochastic processes; multidimensional geometric Brownian motion; DCC-GARCH models; stock price.
AMS 2010 Mathematics Subject Classification Objects : 60G05; 65C60; 97K80.

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Résumé (Full Abstract in French) L'objectif principal de la modélisation financière est de comprendre la dynamique des fluctuations des prix des actifs au cours du temps. Dans cet article, nous mobilisons deux modèles financiers - le mouvement brownien géométrique multidimensionnel et le modèle DCC-GARCH — afin d'analyser ces évolutions. À partir de données de prix quotidiennes de trois banques cotées à la Bourse de Johannesburg, nous estimons les paramètres de chaque modèle et évaluons leur capacité à prévoir les rendements et les prix. Les modèles sont ensuite comparés quant à leur efficacité à simuler les rendements et à prédire les fluctuations des prix, au moyen de critères d'évaluation standards.

Presentation of authors.

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1. Introduction

Understanding the dynamics of multiple asset prices over time in financial markets is important for making investment decisions, risk management, and portfolio construction. Geometric Brownian Motion (GBM), known for its mathematical ease and serving as foundation of the Black-Scholes model for option pricing, is one of the classical models used to characterize asset prices [Contreras *et al.* (2015)]. GBM can be extended to multidimensions to model multiple assets under constant correlations in particularly is convenient when studying portfolios rather than individual assets [Chen *et al.* (2019)]. One of the major limitations of the GBM model is its assumption of constant volatility and correlation which does not conform to real-world financial data.

In reality, volatility levels in financial markets and correlations between assets shift over time, particularly during periods of economic stress and instability [Karatzas *et al.* (1998)]. Consequently, we need models that can adjust to these changes. The Dynamic Conditional Correlation - Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH) model offers such flexibility by allowing both volatility and correlation to vary over time based on historical data [Engle (2002), Engle III and Sheppard (2001)]. This model helps describe the complex and changing relationships between different assets over time.

The main goal of this paper is to compare the two models for asset price movements. This comparison is not only meant to improve our understanding of how markets behave under different conditions, but also has practical applications in areas such as portfolio risk assessment, hedging strategies, and financial forecasting. The motivation behind this study is to bridge the gap between theoretical models and the unpredictable nature of real financial markets, offering tools that are both grounded in mathematics and actual market behavior. Modeling the joint dynamics of asset prices has long been a central challenge in quantitative finance, especially in the areas of portfolio theory, asset pricing, and risk management [Bollerslev *et al.* (1988), Engle *et al.* (1990)].

A widely used approach for capturing the stochastic behavior of asset prices is the GBM. Although the single-asset GBM forms the backbone of the classical Black-Scholes model, its extension to multiple dimensions allows for the simultaneous modeling of several assets with interdependent dynamics. In this setting, asset prices evolve as correlated stochastic processes, driven by Brownian motions with a predefined covariance structure [Chen *et al.* (2019)]. This correlation structure, typically assumed to be constant, plays a critical role in portfolio risk modeling and derivative pricing. Kurniawan and Nuraini were among the first to emphasize the practical value of the Multidimensional GBM (MGBM) model by applying it to the Indonesian stock data. Their results confirmed that modeling assets jointly, rather than separately, improves forecast accuracy, reflecting better real market behavior [Di Asih and Trimono (2018)].

Similarly, Contreras, Llanquihuén, and Villena explored the geometric structure of the multivariate Black-Scholes model and demonstrated how correlations influence the distribution of asset prices through the eigenvalues of the covariance matrix. Their analysis underlined the mathematical importance of properly specifying the correlation matrix to capture the true joint distribution of asset returns [Contreras *et al.* (2015)]. At the core of MGBM lies the representation of correlated Brownian Motions (BM). Chen, Cheng, and Yang proposed a method to decompose correlated BM into independent components using linear transformations. This decomposition provides a clearer understanding of the interdependence among assets and simplifies simulation and analytical techniques in high-dimensional settings [Chen *et al.* (2019)]. A similar line of work by John and Wu introduced dynamic correlation estimators for pairs of Brownian and GBM, thereby addressing the estimation of time-varying relationships between assets, even when the GBM framework itself assumes constant correlations [John and Wu (2022)].

Beyond modeling, MGBM has been widely used in derivative pricing. Brigo, Rapisarda, and Sridi proposed the Multivariate Mixture Dynamics model as an arbitrage-free generalization of the GBM, capable of reproducing market-observed phenomena such as volatility smiles and correlation skews while maintaining consistency across single-asset and index options. This model extends the classic GBM by introducing more flexible marginal and joint distributions while retaining analytical tractability [Brigo *et al.* (2013)]. Other researchers have applied variations of the GBM model to practical problems in finance. Bufalo, Liseo, and Orlando used skewed versions of the GBM process to develop optimal tracking portfolios. Their empirical findings suggest that considering skewness and correlation leads to better portfolio replication, especially in emerging markets [Bufalo *et al.* (2022)]. Meanwhile, Remillard and Rubenthaler extended the GBM framework to a regime-switching context, where asset dynamics shift according to a Markov process. Their regime-based model provided new insights into hedging and pricing in incomplete markets, suggesting that fixed correlation GBM are valuable but may require augmentation in turbulent environments [Remillard and Rubenthaler (2023)].

In risk management applications, the importance of understanding correlations cannot be overstated. Mankilik and Ihedioha incorporated the GBM process in insurance models, highlighting its flexibility in capturing the joint movement of investment portfolios and claims reserves [Mankilik and Ihedioha (2024)]. Together, these studies demonstrate that while the assumption of constant correlation in MGBM simplifies analysis, it remains a powerful. It captures essential market dynamics, supports a broad array of financial applications, and provides a springboard for more advanced modeling techniques.

Volatility is a key concept in financial econometrics, often used as a measure of risk and uncertainty associated with asset returns. During the early development of financial models, volatility was frequently assumed to be constant over time. However, empirical studies have shown that this assumption does not hold in real financial markets. Instead, asset returns tend to exhibit volatility clustering, where periods of high volatility are followed by high volatility and periods of low volatility are followed by low volatility. This observed behavior has led to the development of more advanced models that account for time-varying volatility. Among these, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models introduced by Bollerslev in 1986 have become particularly influential [Engle III and Sheppard (2001), Silvennoinen and Teräsvirta (2009)]. These models extend earlier work by Engle, in 1982, on Autoregressive Conditional Heteroskedasticity (ARCH) models and provide a more flexible framework for modeling volatility dynamics.

GARCH models are widely used in both academic research and practical applications, including risk management, portfolio optimization, and derivative pricing, due to their ability to capture the stylized facts of financial time series. The modeling of time-varying volatility began with the ARCH model introduced by Engle [Engle (1982)], which allowed conditional variances to be modeled as functions of past squared errors. Bollerslev extended this model with the GARCH model, incorporating both past errors and past variances, and thus enabling a more accurate representation of persistent volatility over time [Bollerslev (1986)]. These univariate models are effective for analyzing single asset volatility, but they fall short when applied to portfolios involving multiple assets. The need to capture co-movement and dependencies among multiple financial assets led to the development of multivariate extensions of GARCH models. One of the earliest models in this category was the Constant Conditional Correlation (CCC) model developed by Bollerslev [Bollerslev (1990)], which assumes constant correlations across time. While computationally simple, this assumption is overly restrictive in real-world scenarios where correlations among asset returns fluctuate over time.

The Dynamic Conditional Correlation (DCC) model, introduced by Engle and Sheppard, addressed the limitations of CCC by allowing the correlation matrix to change over time. The DCC-GARCH model belongs to a class of models that separately estimate time-varying variances and correlations, thus allowing for greater flexibility while maintaining computational efficiency [Nordström (2021)]. In the DCC-GARCH model, the conditional covariance matrix is decomposed into a diagonal matrix of standard deviations and a time-varying correlation matrix. This modular structure ensures that the number of parameters to estimate remains manageable even as the number of assets increases. This makes the DCC-GARCH particularly suitable for large-scale portfolio modeling and forecasting [Engle (2002), Engle III and Sheppard (2001)].

A key factor influencing the effectiveness of GARCH-type models is the choice of distribution for the error terms. While early implementations often assumed normality, empirical return distributions frequently exhibit heavy tails and skewness. These features cannot be captured adequately by Gaussian distributions, prompting researchers to explore alternative specifications. Orskaug implemented the DCC-GARCH model under Gaussian, Student's t, and skewed Student's t distributions [Blanco and Oks (2004)]. Her results showed that models incorporating heavier tails and asymmetry provided better fit and more accurate Value-at-Risk (VaR) estimates, especially during periods of financial turmoil. Azzalini and Capitanio supported these findings through their development of the multivariate skew-t distribution, which captures both asymmetry and excess kurtosis [Azzalini and Capitanio (2003)]. Additional contributions by Aas, Dimakos, and Haff [Aas *et al.* (2005)] and Venter and de Jongh [Venter and De Jongh (2001)] emphasized the utility of the Normal Inverse Gaussian (NIG) distribution in risk estimation. These distributional improvements allow DCC-GARCH models to better align with the empirical properties of financial data.

The estimation of DCC-GARCH models typically follows a two-step procedure: first, individual univariate GARCH models are estimated for each asset to obtain conditional variances; second, these standardized residuals are used to estimate the time-varying correlation matrix. This stepwise approach improves tractability and ensures that the model remains parsimonious. Evaluation of these models is often conducted through statistical tests and backtesting. Nordström applied various DCC-GARCH specifications and tested their predictive power using real market data. His findings confirmed the superiority of models incorporating non-normal error distributions. Goodness-of-fit tests, such as the Ljung-Box test and autocorrelation analysis, provided insights into the model's performance, while VaR backtesting using Kupiec and Christoffersen tests validated its effectiveness in capturing risk [Nordström (2021)]. Other studies, including those by Peters [Peters (2008)] and Blanco and Oks [Blanco and Oks (2004)], have emphasized the importance of rigorous backtesting. Their findings demonstrate that DCC-GARCH models, when properly specified and calibrated, are highly effective in forecasting risk and volatility.

DCC-GARCH models have a wide array of applications, including portfolio optimization, risk management, and asset allocation. Jondeau explored how relaxing the assumption of normality affects conditional asset allocation. His findings demonstrated the potential costs of using mean-variance criteria under non-normal conditions [Jondeau and Rockinger (2005)]. Almeida and Vicente also highlighted the importance of accounting for interest rate options when assessing interest rate risk, a perspective that reinforces the value of flexible multivariate models in capturing the full spectrum of financial market dynamics [Almeida and Vicente (2009)].

The DCC-GARCH model is underpinned by solid statistical theory. Tsay [Tsay (2005)], and Cryer and Chan [Cryer (2008)] offered comprehensive treatments of time series analysis, equipping practitioners with tools for identifying and estimating model parameters. Anderson provided foundational insights into multivariate statistical analysis, which are essential for understanding the complexities involved in modeling high-dimensional data [Anderson (1958)]. Silvennoinen and Teräsvirta provided a comprehensive survey of multivariate GARCH models, categorizing them into types such as VEC, BEKK, factor models, and models of variances and correlations. Their work situated DCC-GARCH among a wider range of volatility modeling approaches [Silvennoinen and Teräsvirta (2009)].

This paper is organized into five sections. The next section (Section 2) provides the necessary mathematical foundations of the MGBM and DCC-GARCH models and discusses how to estimate the parameters involved in the MGBM and DCC-GARCH models. Section 3 presents an empirical comparison and application of the models, including a description of the dataset and the preprocessing steps involved. Section 4 introduces the metrics used to evaluate the performance of both models and discusses their comparative results. Finally, Section 5 summarizes the key findings of the study and suggests potential directions for future research.

2. Mathematical Foundations of MGBM and DCC-GARCH Models

In this section, we provide an overview of the MGBM and the DCC-GARCH models. We discuss the mathematical foundations of these models and present the theory behind parameter estimation. The aim is to highlight both the stochastic dynamics captured by MGBM and the volatility and correlation structures modeled by DCC-GARCH, thereby setting the stage for their application in financial modeling and analysis. The material presented in this section can be found in [Di Asih and Trimono (2018), Engle III and Sheppard (2001), Paul Glasserman (2003), Zilinskas (2004)] and the references therein.

2.1. ***MGBM Model***

The GBM has long been a cornerstone in financial modelling due to its analytical tractability and its role in the Black-Scholes model. MGBM extends the classical concept of GBM to a higher-dimensional setting, allowing for the simultaneous modeling of multiple assets whose dynamics may be correlated. This extension is particularly useful in financial applications, where the joint behavior of several asset prices must be captured to account for dependencies and co-movements among them.

Let $S_t = (S_t^1, \dots, S_t^d)$ be a d-dimensional process. We say that S_t follows a MGBM with drift vector $\boldsymbol{\mu} = (\mu_1, \dots, \mu_d)^\top \in \mathbb{R}^d$ and covariance matrix $\boldsymbol{\Sigma} = [\sigma_i \sigma_j \rho_{ij}] \in \mathbb{R}^{d\times d}$ if each component satisfies the following equation

$$\frac{dS_i(t)}{S_i(t)} = \mu_i \, dt + \sigma_i \, dX_i(t), \quad \text{for} \quad i = 1, \dots, d, \tag{1}$$

where $\sigma_i > 0$ is the volatility of asset i, $X_i(t)$ are correlated Brownian motion such that $\mathbb{E}[X_i(t)X_j(t)] = \rho_{ij} t$, with covariance matrix $\Sigma = [\sigma_i \sigma_j \rho_{ij}]$ of the vector process $(X_1(t), \dots, X_d(t))$.

The solution for each asset is

$$S_i(t) = S_i(0) \exp\left[\left(\mu_i - \tfrac{1}{2}\sigma_i^2\right) t + \sigma_i X_i(t)\right]. \tag{2}$$

Moreover, by a convenient abuse of terminology, we refer to $\boldsymbol{\mu}$ as the drift vector of S_t, $\boldsymbol{\Sigma}$ as its covariance matrix, and $\boldsymbol{R}$ as its correlation matrix with entries ρ_{ij} . The instantaneous drift of S_t is $\mu_i S_i(t)$, and the instantaneous covariance between $S_i(t)$ and $S_j(t)$ is

$$\text{Cov}[S_i(t), S_j(t)] = S_i(0)S_j(0)\exp((\mu_i + \mu_j)t)\left(e^{\rho_{ij}\sigma_i\sigma_j t} - 1\right). \tag{3}$$

The MGBM model allows for the simultaneous modelling of multiple assets through a fixed correlation structure. However, its practical use depends on accurate estimation of the drift vector, the volatility vector , and the correlation matrix.

2.2. ***Parameter Estimation for MGBM***

The model presented in equation (2) involves several unknown parameters: the drift vector $\boldsymbol{\mu}$, the volatility vector $\boldsymbol{\sigma}$, and the correlation matrix $\mathbf{R}$. To estimate these parameters, we apply the Method of Maximum Likelihood (MML), under the assumption that the observations are taken at regular time intervals of length Δt. Note that the returns are log-normally distributed, we then first derive their joint distribution.

Suppose that S is a d-dimensional GBM with parameters $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$.The log-returns $\mathbf{r}_i = (r_{i1}, \ldots, r_{id})^\top$, where

$$r_{ij} = \log\left(\frac{S_j(i\Delta t)}{S_j((i-1)\Delta t)}\right), \tag{4}$$

are independent and identically distributed, with $r_i \sim \mathcal{N}_d(\mathbf{m}\Delta t, \boldsymbol{\Sigma}\Delta t)$, where

$$m_j = \mu_j - \frac{1}{2}\sigma_j^2, \quad \text{and} \quad \boldsymbol{\Sigma} = \text{diag}(\boldsymbol{\sigma})\mathbf{R}\text{diag}(\boldsymbol{\sigma}) = [\sigma_i\sigma_j\rho_{ij}].$$

It is worth noting that the maximum likelihood estimates are obtained from the sample mean and covariance of the returns.

Let define the empirical sample mean and sample covariance matrix of the log-returns as follows

$$\bar{\mathbf{r}} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{r}_i, \quad \mathbf{S}_r = \frac{1}{n}\sum_{i=1}^{n}(\mathbf{r}_i - \bar{\mathbf{r}})(\mathbf{r}_i - \bar{\mathbf{r}})^\top. \tag{5}$$

Given that the returns r_i are independent and identically distributed Gaussian vectors with $r_i \sim \mathcal{N}_d(\mathbf{m}\Delta t, \boldsymbol{\Sigma}\Delta t)$. The joint likelihood function is given by

$$L(\boldsymbol{\theta}) = \prod_{i=1}^{n}\frac{1}{(2\pi\Delta t)^{d/2}|\boldsymbol{\Sigma}|^{1/2}}\exp\left\{-\frac{1}{2\Delta t}(\mathbf{r}_i - \mathbf{m}\Delta t)^\top\boldsymbol{\Sigma}^{-1}(\mathbf{r}_i - \mathbf{m}\Delta t)\right\}, \tag{6}$$

where $\boldsymbol{\theta}$ denotes the set of model parameters. By taking the logarithm of equation (6), the log-likelihood function becomes

$$\log(L(\boldsymbol{\theta})) = \log\left(\prod_{i=1}^{n}\frac{1}{(2\pi\Delta t)^{d/2}|\boldsymbol{\Sigma}|^{1/2}}\exp\left\{-\frac{1}{2\Delta t}(\mathbf{r}_i - \mathbf{m}\Delta t)^\top\boldsymbol{\Sigma}^{-1}(\mathbf{r}_i - \mathbf{m}\Delta t)\right\}\right). \tag{7}$$

By minimizing the log-likelihood function, we obtained the Maximum Likelihood Estimators (MLEs) of the model parameters, which are

$$\hat{\boldsymbol{\Sigma}} = \frac{\mathbf{S}_r}{\Delta t}, \quad \hat{\mathbf{m}} = \frac{\bar{\mathbf{r}}}{\Delta t}. \tag{8}$$

Using equation (5) together with equation (8), we derive the maximum likelihood estimates for μ_j, σ_j, and ρ_{jk} corresponding to the model given in equation (2). Their expressions are given as follows

$$\hat{\mu}_j = \frac{\bar{r}_j}{\Delta t} + \frac{(\mathbf{S}_r)_{jj}}{2\Delta t}, \quad \hat{\sigma}_j = \sqrt{\frac{(\mathbf{S}_r)_{jj}}{\Delta t}}, \quad \hat{\rho}_{jk} = \frac{(\mathbf{S}_r)_{jk}}{\Delta t \hat{\sigma}_j \hat{\sigma}_k}, \tag{9}$$

for $j, k \in \{1, \ldots, d\}$.

2.3. ***DCC-GARCH Model***

2.3.1. **Univariate GARCH Model**

In traditional econometric models, it was usually assumed that the variance over one period is constant. However, in 1982, Robert F. E. introduced the Autoregressive Conditional Heteroscedasticity (ARCH) model, which allows the conditional variance to depend on past errors. In the ARCH process, although the conditional variance changes over time, the unconditional variance remains constant. ARCH processes are typically zero-mean, serially uncorrelated time series, but their variance varies depending on past information [Engle (1982), Bollerslev *et al.* (1994)].

The ARCH model of order q is defined as follows

$$\begin{aligned} y_t &= \epsilon_t \sqrt{h_t}, \\ h_t &= \alpha_0 + \sum_{i=1}^{q} \alpha_i y_{t-i}^2, \end{aligned} \tag{10}$$

where y_t is a zero-mean stochastic process, ϵ_t is a white noise process with mean zero and variance one, and h_t is the conditional variance of y_t given past information. The model parameters satisfy $\alpha_0 > 0$ and $\alpha_i \geq 0$ for $i = 1, \ldots, q$.

In 1986, Bollerslev generalized the ARCH model by introducing the GARCH model, which is better at predicting volatilities compared to the ARCH model. The extension from the ARCH model to the GARCH model allows past conditional variances to enter the current conditional variance equation, providing greater flexibility and improved modeling of financial time series [Bollerslev (1986)].

Mathematically, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, denoted by GARCH(p, q), is defined as follows

$$\begin{aligned} r_t &= \mu_t + y_t, \\ y_t &= \sqrt{h_t}\, z_t, \\ h_t &= \alpha_0 + \sum_{i=1}^{q} \alpha_i y_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}, \end{aligned} \tag{11}$$

where r_t is the log-return at time t, μ_t is the conditional mean of r_t (often assumed constant), y_t is the residual return (shock), z_t is an i.i.d. sequence with $\mathbb{E}[z_t] = 0$ and Var$[z_t] = 1$, h_t is the conditional variance of y_t, $\alpha_0 > 0$, $\alpha_i \geq 0$, and $\beta_j \geq 0$ for all i, j, p and q are the GARCH orders. Note that the term $\sum_{i=1}^{q} \alpha_i y_{t-i}^2$ represents the ARCH component, while $\sum_{j=1}^{p} \beta_j h_{t-j}$ represents the GARCH component. To ensure covariance stationarity, it is required that $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$.

2.3.2. **DCC-GARCH models**

The GARCH family of models was introduced to account for the empirical observation that asset returns exhibit persistent volatility. In its univariate form, a GARCH(p,q) model specifies the conditional variance of returns as a function of past squared returns and past conditional variances, thereby capturing volatility clustering. To analyze portfolios or multiple assets jointly, multivariate extensions of GARCH models are required. The Constant Conditional Correlation (CCC-GARCH) model assumes correlations between assets are fixed, an assumption often too restrictive. The DCC-GARCH belongs to the class of multivariate GARCH models that capture time-varying conditional variances and correlations. It allows both the conditional variances and correlations to evolve dynamically [Bollerslev *et al.* (1994), Long and Ullah (2005), Silvennoinen and Teräsvirta (2009), Van der Weide (2002)].

Suppose that we have log returns $\mathbf{r}_t$ from n financial assets, with conditional expected value $\boldsymbol{\mu}_t$ and covariance matrix $\mathbf{H}_t$. The DCC-GARCH model is defined as

$$\begin{aligned} \mathbf{r}_t &= \boldsymbol{\mu}_t + \mathbf{y}_t, \\ \mathbf{y}_t &= \mathbf{H}_t^{1/2} \boldsymbol{z}_t, \\ \mathbf{H}_t &= \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t, \end{aligned} \tag{12}$$

where $\mathbf{r}_t$ is the log returns of n assets at time t, an $n \times 1$ vector, $\boldsymbol{\mu}_t$ is the $n \times 1$ conditional mean vector of $\mathbf{r}_t$, $\mathbf{y}_t$ is the mean-corrected returns, that is, $\mathbb{E}[\mathbf{y}_t] = \mathbf{0}$ and $\mathrm{Cov}[\mathbf{y}_t] = \mathbf{H}_t$, $\mathbf{H}_t$ is the $n \times n$ conditional covariance matrix of $\mathbf{y}_t$, matrix, $\mathbf{D}_t$ is the $n \times n$ diagonal matrix of conditional standard deviations of $\mathbf{y}_t$, $\mathbf{R}_t$ is the $n \times n$ conditional correlation matrix of $\mathbf{y}_t$, and $\boldsymbol{z}_t$ is the vector of i.i.d. errors such that $\mathbb{E}[\boldsymbol{z}_t] = \mathbf{0}$ and $\mathbb{E}[\boldsymbol{z}_t \boldsymbol{z}_t^\top] = \mathbf{I}$.

It is worth to note that the matrix $\mathbf{D}_t$ can be expressed as

$$\mathbf{D}_t = \begin{pmatrix} \sqrt{h_{1t}} & 0 & \cdots & 0 \\ 0 & \sqrt{h_{2t}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sqrt{h_{nt}} \end{pmatrix}, \tag{13}$$

with each variance h_{it} modeled by a univariate GARCH process. Typically, a GARCH(1,1) model is used, which is defined as

$$h_{it} = \alpha_{i0} + \sum_{j=1}^{q_i} \alpha_{ij} y_{i,t-j}^2 + \sum_{j=1}^{p_i} \beta_{ij} h_{i,t-j}, \quad i = 1, \ldots, n. \tag{14}$$

This modeling ensures that the individual volatilities of the asset returns are captured accurately. The standardized residuals are given by

$$\boldsymbol{\epsilon}_t = \mathbf{D}_t^{-1} \mathbf{y}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t). \tag{15}$$

The matrix $\mathbf{R}_t$ is the time-varying conditional correlation matrix of $\boldsymbol{\epsilon}_t$ and is symmetric, expressed as

$$\mathbf{R}_t = \begin{pmatrix} 1 & \rho_{12,t} & \rho_{13,t} & \cdots & \rho_{1n,t} \\ \rho_{12,t} & 1 & \rho_{23,t} & \cdots & \rho_{2n,t} \\ \rho_{13,t} & \rho_{23,t} & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \rho_{(n-1)n,t} \\ \rho_{1n,t} & \rho_{2n,t} & \cdots & \rho_{(n-1)n,t} & 1 \end{pmatrix}. \tag{16}$$

Each off-diagonal element $\rho_{ij,t}$ captures the dynamic correlation between standardized residuals $\boldsymbol{\epsilon}_{it}$ and $\boldsymbol{\epsilon}_{jt}$. Consequently, each element of the conditional covariance matrix $\mathbf{H}_t$ can be written as

$$[\mathbf{H}_t]_{ij} = \sqrt{h_{it} h_{jt}} \rho_{ij,t}, \quad \text{with} \quad \rho_{ii,t} = 1. \tag{17}$$

This structure guarantees that $\mathbf{H}_t$ is positive definite if each h_{it} is strictly positive and $\mathbf{R}_t$ is positive definite. The key innovation of the DCC-GARCH model is the dynamic evolution of $\mathbf{R}_t$, it has a dynamic conditional correlation structure of the form

$$\begin{aligned} \mathbf{R}_t &= \mathbf{Q}_t^{*-1} \mathbf{Q}_t \mathbf{Q}_t^{*-1}, \\ \mathbf{Q}_t &= (1 - a - b)\bar{\mathbf{Q}} + a \boldsymbol{\epsilon}_{t-1} \boldsymbol{\epsilon}_{t-1}^T + b \mathbf{Q}_{t-1}, \end{aligned} \tag{18}$$

where $\bar{\mathbf{Q}} = \mathrm{Cov}[\boldsymbol{\epsilon}_t \boldsymbol{\epsilon}_t^T] = \mathbb{E}[\boldsymbol{\epsilon}_t \boldsymbol{\epsilon}_t^T]$ is the unconditional covariance matrix of the standardized residuals, $a \geq 0$ and $b \geq 0$ are scalar parameters controlling the impact of recent shocks and past correlation dynamics, respectively, $a + b < 1$ ensures stationarity and positive definiteness of the process.

The matrix $\bar{\mathbf{Q}}$ is estimated empirically from the data as

$$\bar{\mathbf{Q}} = \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{\epsilon}_t \boldsymbol{\epsilon}_t^T. \tag{19}$$

To ensure that $\mathbf{R}_t$ is a valid correlation matrix, the scaling matrix $\mathbf{Q}_t^*$ is defined as a diagonal matrix with the square root of the diagonal elements of $\mathbf{Q}_t$ on the diagonal

$$\mathbf{Q}_t^* = \begin{pmatrix} \sqrt{q_{11t}} & 0 & \dots & 0 \\ 0 & \sqrt{q_{22t}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \sqrt{q_{nnt}} \end{pmatrix}. \tag{20}$$

Thus, the elements of $\mathbf{R}_t$ are computed as follows

$$|\rho_{ij}| = \left| \frac{q_{ijt}}{\sqrt{q_{iit}q_{jjt}}} \right| \leq 1.$$

This standardization ensures that $\mathbf{R}_t$ has unit diagonal and remains a valid correlation matrix at each time step.

2.3.3. **Parameter Estimation for DCC-GARCH**

This section outlines the procedure for estimating the parameters of the GARCH (DCC-GARCH) model. In this study, we assume that the standardized errors follow a multivariate Gaussian distribution. Given that the standardized errors $\boldsymbol{z}_t$ are independently and identically distributed as multivariate normal, the joint density of $\boldsymbol{z}_1, \dots, \boldsymbol{z}_T$ is given by

$$f(\boldsymbol{z}_t) = \prod_{t=1}^{T} \frac{1}{(2\pi)^{n/2}} \exp\{-\frac{1}{2}\boldsymbol{z}_t^T \boldsymbol{z}_t\}. \tag{21}$$

Let $\boldsymbol{r}_t$ denote the n-dimensional return vector at time t, with conditional mean vector $\boldsymbol{\mu}_t$ and conditional covariance matrix $\mathbf{H}_t$, as defined in the multivariate GARCH framework. If $\boldsymbol{r}_t \sim \mathcal{N}(\boldsymbol{\mu}_t, \mathbf{H}_t)$, then the mean-corrected returns $\boldsymbol{y}_t = \boldsymbol{r}_t - \boldsymbol{\mu}_t$ are distributed as $\boldsymbol{y}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{H}_t)$. Furthermore, standardized errors $\boldsymbol{z}_t = \mathbf{H}_t^{-1/2}\boldsymbol{y}_t$ follow a standard multivariate normal distribution, that is, $\boldsymbol{z}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_n)$. The likelihood function for the sample $\{\boldsymbol{y}_t\}_{t=1}^T$ is therefore given by

$$L(\boldsymbol{\theta}) = f(\boldsymbol{y}_t \mid \boldsymbol{\theta}) = \prod_{t=1}^{T} \frac{1}{(2\pi)^{n/2}|\mathbf{H}_t|^{1/2}} \exp\{-\frac{1}{2}\boldsymbol{y}_t^T \boldsymbol{H}_t^{-1}\boldsymbol{y}_t\}, \tag{22}$$

where $\boldsymbol{\theta}$ denotes the set of model parameters, and $|\mathbf{H}_t|$ denotes the determinant of $\mathbf{H}_t$. Taking the logarithm and using the decomposition $\mathbf{H}_t = \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t$, the log-likelihood function becomes

$$\log(L(\boldsymbol{\theta})) = -\frac{1}{2}\sum_{t=1}^{T} \left(n\log(2\pi) + 2\log(|\mathbf{D}_t|) + \log(|\mathbf{R}_t|) + \boldsymbol{y}_t^T \mathbf{D}_t^{-1}\mathbf{R}_t^{-1}\mathbf{D}_t^{-1}\boldsymbol{y}_t\right). \tag{23}$$

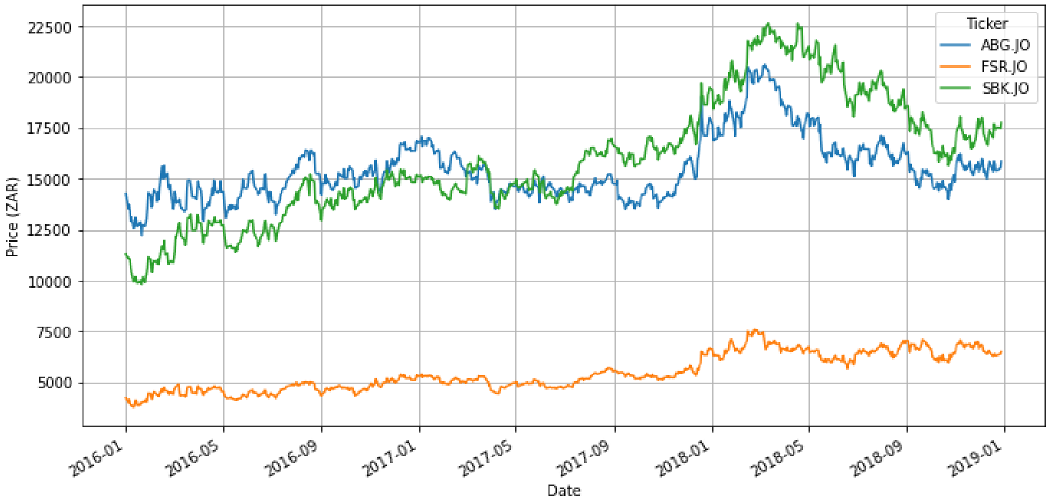


Fig. 1: Daily closing prices of ABG.JO, FSR.JO, SBK.JO.

Let the parameter vector $\boldsymbol{\theta}$ be partitioned into two groups: $(\phi, \psi) = (\phi_1, \ldots, \phi_n, \psi)$, where each $\phi_i = (\alpha_{0i}, \alpha_{1i}, \ldots, \alpha_{qi}, \beta_{1i}, \ldots, \beta_{pi})$ contains the parameters of the univariate GARCH model for the i-th asset series. The vector $\psi = (a, b)$ contains the parameters of the correlation structure in equation (18).

To maximize the likelihood, we proceed in two steps. In the first step, we estimate the parameters ϕ of the univariate GARCH models for each asset series, replacing $\mathbf{R}_t$ with the identity matrix $\mathbf{I}_n$. In the second step, we estimate the parameters ψ of the correlation structure using the correctly specified log-likelihood in equation (23), given the estimated ϕ.

3. Empirical Comparison and Applications

3.1. *Data Description*

The dataset used in this study consists of the daily closing prices of three major banking companies listed on the Johannesburg Stock Exchange (JSE): Absa Group Limited (ABG.JO), FirstRand Limited (FSR.JO), and Standard Bank Group Limited (SBK.JO). The sample period covers 01 January 2016 to 01 January 2020, with a total of 1016 observations obtained from Yahoo Finance. Missing observations are adjusted using forward filling from the previous day's trading. Figure 1 shows the daily closing prices of the three companies during this period.

The daily returns of market i at time t are given by

$$r_{it} = \log(x_{i,t}) - \log(x_{i,t-1}), \tag{24}$$

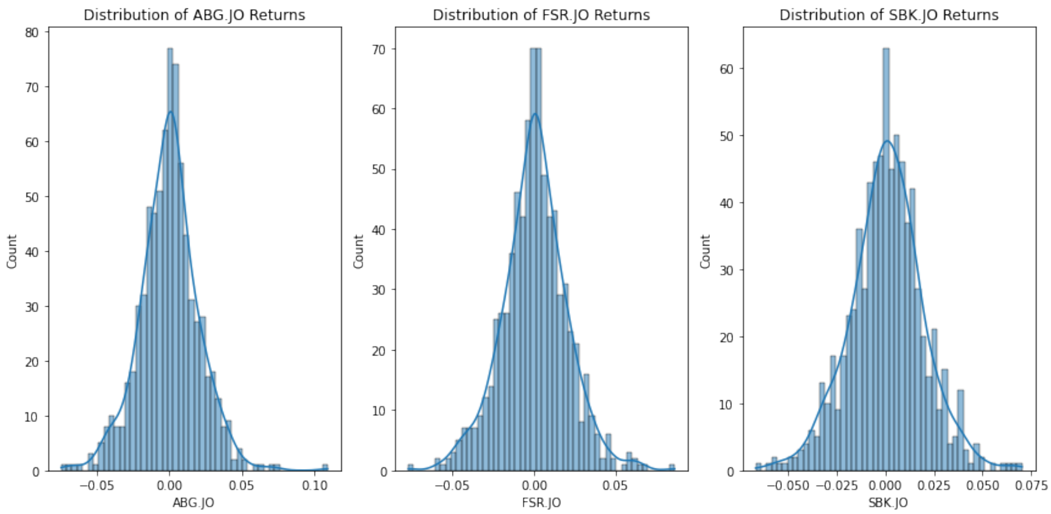


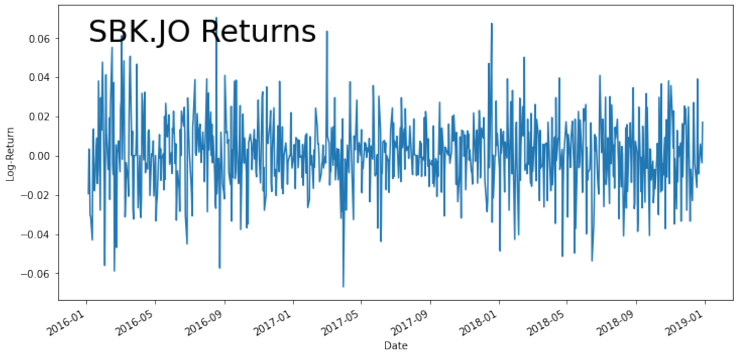
Fig. 2: Distribution of ABG.JO, FSR.JO, SBK.JO.

Table 1: Descriptive Statistics of Stock Market Indices Returns.

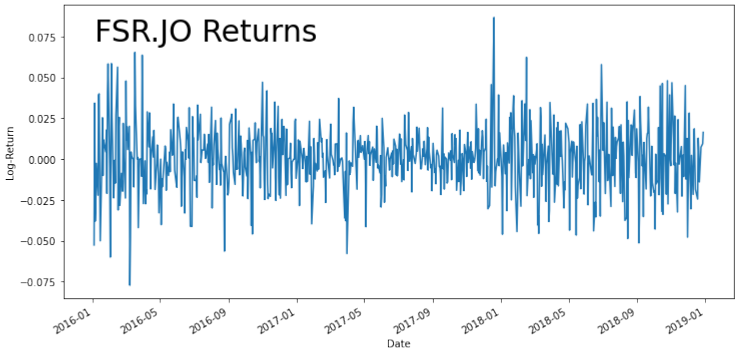
	Mean	**Std. Dev.**	**Skewness**	**Kurtosis**	**ADF Test**	**K-S Stat**	**K-S p-value**
ABG.JO	0.000139	0.020291	0.135131	1.717962	0.0	0.047783	0.058848
FSR.JO	0.000567	0.020050	0.064895	1.117497	0.0	0.039694	0.174684
SBK.JO	0.000592	0.019000	0.039904	0.820162	0.0	0.036370	0.257420

where $x_{i,t}$ is the daily closing price of the i-th market. Figure 3 illustrates the corresponding log-returns for the three companies. The Figure 2 shows the distribution of the log-returns of the three companies.

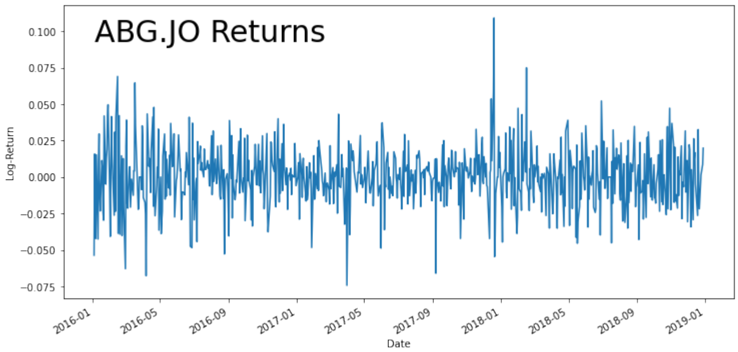
We chose to use the data from January 01, 2016, to December 31, 2018, to train our models, and data from January 01, 2019, to January 01, 2020, for testing. The Table 1 shows some basic distributional properties of the daily log-returns for each company's stock during the training data. From the table, we can observe that the stock prices selected have diverse returns and volatility. The average returns of the three companies are positive and around zero. In terms of volatility, ABG.JO shows the highest variation, while SBK.JO has the lowest. The skewness values for ABG.JO, FSR.JO, and SBK.JO are all close to zero, which means that their return distributions are approximately symmetric with no significant skew. To check the stationarity of the series, we applied the Augmented Dickey-Fuller (ADF) test. The results showed that all the series are stationary, as p-values were less than 0.05 for each company. We also used the Kolmogorov-Smirnov test normality to check the normality of the data. Based on the results in Table 1, we could not reject the null hypothesis. Therefore, we can conclude that the stock prices for these three companies are approximately normally distributed.



(a) Daily log-returns of SBK.JO



(b) Daily log-returns of FSR.JO



(c) Daily log-returns of ABG.JO

Fig. 3: Daily log-returns of the three JSE-listed banks.

3.2. *Estimation of the parameters for MGBM*

This section presents the estimated parameters of the MGBM model based on the historical return data, using the mathematical approach explained in Section 2. The drift $\hat{\mu}$ and volatility $\hat{\sigma}$ for each asset were estimated using maximum likelihood,

Table 2: Drift and Volatility estimations.

	ABG.JO	**FSR.JO**	**SBK.JO**
$\hat{\mu}$	0.144611	0.149150	0.064017
$\hat{\sigma}$	0.289949	0.304075	0.314164

Table 3: Correlation Matrix $\hat{\boldsymbol{R}}$.

	ABG.JO	**FSR.JO**	**SBK.JO**
ABG.JO	1.000000	0.788262	0.758052
FSR.JO	0.788262	1.000000	0.742334
SBK.JO	0.758052	0.742334	1.000000

Table 4: Covariance Matrix $\hat{\Sigma}$.

	ABG.JO	**FSR.JO**	**SBK.JO**
ABG.JO	0.084070	0.069498	0.069052
FSR.JO	0.069498	0.092462	0.070915
SBK.JO	0.069052	0.070915	0.098699

as given in equation (9), and the results are reported in Table 2. The results indicate that FSR.JO exhibits the highest annualized drift (14.92%) and SBK.JO shows the highest volatility (31.42%), while ABG.JO demonstrates the most favorable risk-return profile with moderate drift (14.46%) and the lowest volatility (28.99%). The interdependence between assets is captured through correlation and covariance matrices shown in Tables 3 and 4, respectively. All three stocks show strong positive correlations, with values between 0.74 and 0.79. The strongest link is between ABG.JO and FSR.JO, which have a correlation of 0.79. The covariance matrix reflects both the strength of these relationships and the individual volatilities of the assets. The highest covariance is between SBK.JO and itself (0.0987), which is expected due to its high volatility.

3.3. ***DCC-GARCH Model***

This section presents the estimated parameters of the DCC-GARCH model, using the two-stage approach explained in Section 2. The results are divided into two parts: the first part includes univariate GARCH(1,1) models for each asset, and the second part presents the dynamic correlation parameters.

1. **First-stage estimation of the univariate GARCH parameters.** The parameter α_0, which represents the base level of volatility, ranges from 0.000008 to 0.000034 across stocks, with ABG.JO having the highest baseline volatility. The parameter α_1, which captures immediate shock responses, lies between 0.048 and 0.075.This suggests a moderate sensitivity to recent market movements. The parameter β_1, which shows how much current volatility depends on past

Table 5: First-Stage Estimated Coefficients.

	ABG.JO	FSR.JO	SBK.JO
α_0	0.000034	0.000008	0.000009
α_1	0.075424	0.050000	0.048147
β_1	0.839133	0.929999	0.925650
$\alpha_1 + \beta_1$	0.914557	0.979999	0.973797

Table 6: DCC Parameter Estimates.

Parameter	Value	Interpretation
a	0.035210	Low immediate reaction to shocks.
b	0.804352	Strong memory of past correlations.
$a + b$	0.839562	High but stable long-term dependence.

volatility, is quite high, ranging from 0.84 to 0.93. This indicates strong persistence in volatility. The sum $\alpha_1 + \beta_1$ is between 0.915 and 0.980, confirming that the volatility in all three stocks is highly persistent but still stable. Among them, ABG.JO exhibits the strongest reaction to news ($\alpha_1 = 0.075$) but the lowest persistence, while FSR.JO and SBK.JO show more stable volatility patterns with greater dependence on historical values.

2. **Second-stage estimation of DCC Parameters.** The DCC parameters that describe how correlations change over time show clear patterns. Table 6 presents the DCC parameters. The news sensitivity parameter ($a = 0.035$) indicates correlations respond gradually to new information, while the memory parameter ($b = 0.804$) reveals strong dependence on historical correlations. The sum of these two parameters (0.840) confirms that the correlation structure is highly persistent.This means that while individual stock volatilities respond quickly to market events, the relationships between the stocks tend to change more slowly, remaining relatively stable over time.

These results show that individual stock volatilities react quickly to news, sometimes within a day, but the relationships between stocks change more slowly. This is important for managing risk because both the fast changes in volatility and the slower changes in correlations need to be considered when creating portfolios.

4. Results and Discussions

This section investigates how well the two models perform using stock price data from three companies listed on the Johannesburg Stock Exchange. Using the estimated parameters from earlier, we evaluate the models on data from January 1, 2019, to January 1, 2020.

4.1. *Error checking*

Firstly, let us describe the metrics that we are going to use to measure their performances.

1. **Mean Absolute Error (MAE)** measures the average magnitude of errors between the actual values and the corresponding predicted values, without considering whether the errors are positive or negative. It is calculated as follows:
$$\mathrm{MAE}(C_k) = \frac{1}{n}\sum_{i=0}^{n-1} |A(C_k)_i - P(C_k)_i|,$$
where $C_k \in \{\text{SBK.JO, FSR.JO, ABG.JO}\}$, $A(C_k)$ represents the actual values, $P(C_k)$ represents the predicted values, and n is the number of observations. Note that a lower MAE indicates better model accuracy.
2. **Root Mean Squared Error (RMSE)** penalizes larger errors more heavily than MAE, making it sensitive to outliers. It is computed as:
$$\mathrm{RMSE}(C_k) = \sqrt{\frac{1}{n}\sum_{i=0}^{n-1} (A(C_k)_i - P(C_k)_i)^2}.$$
RMSE provides a measure of how spread out the residuals are, with lower values indicating better predictive performance.
3. **Mean Absolute Percentage Error (MAPE)** expresses the error as a percentage of the actual values, making it a relative measure. It is defined as:
$$\mathrm{MAPE}(C_k) = \frac{100}{n}\sum_{i=0}^{n-1} \left|\frac{A(C_k)_i - P(C_k)_i}{A(C_k)_i}\right|.$$
MAPE is useful for comparing forecast accuracy across different datasets, with lower percentages indicating higher precision.

4.2. *Performance of MGBM*

For forecasting returns, we use MAE and RMSE to measure accuracy on the test set's log-returns. The model performs differently across the three stocks. SBK.JO has the largest errors, with an MAE of 0.0259 and RMSE of 0.0010, meaning its returns are the hardest to predict. FSR.JO has smaller errors, with an MAE of 0.0166 and RMSE of 0.0004. ABG.JO has the smallest errors, with an MAE of 0.0159 and RMSE of 0.0004, making it the most predictable. Figure 4 shows the forecasted returns compared to the actual returns for each day and stock. Table 7 lists the exact error values and confirms that while the model works fairly well, some stocks are easier to predict than others.

For price predictions, we use MAPE to measure how close the forecasted prices are to actual prices in percentage terms. Figure 5 displays the actual and predicted

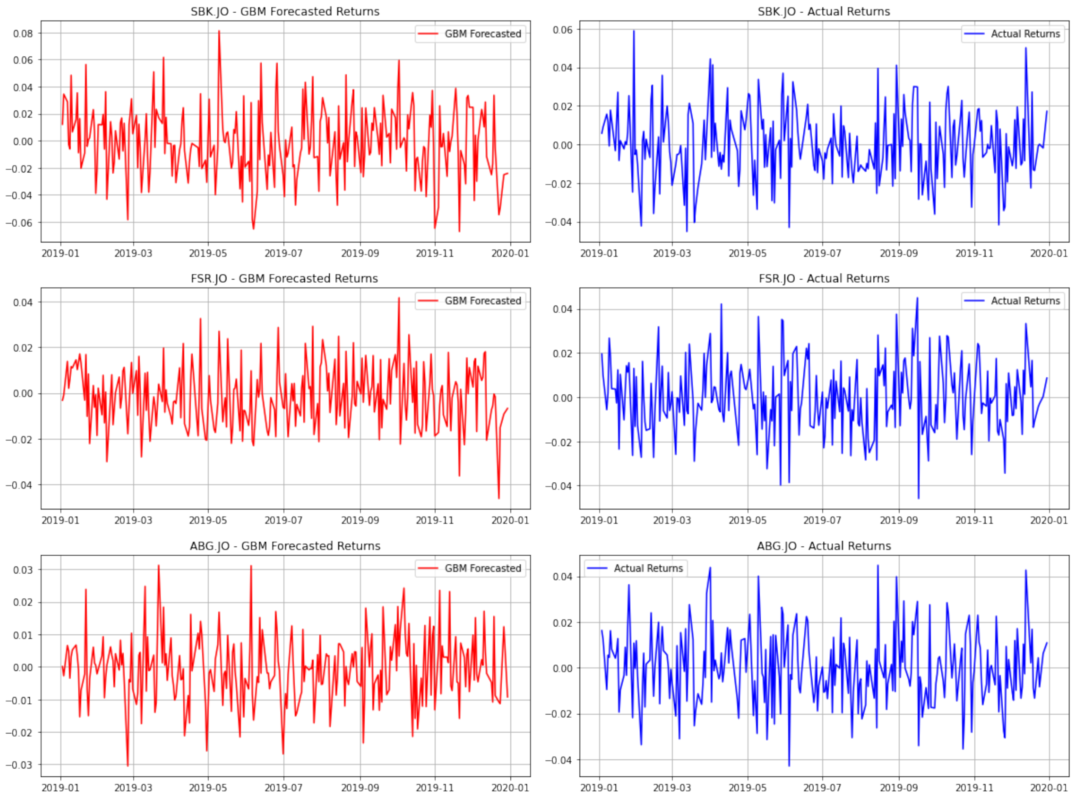


Fig. 4: MGBM forecasted returns.

Table 7: Forecasting returns using MGBM.

	ABG.JO	**FSR.JO**	**SBK.JO**
MAE	0.015933	0.016589	0.025864
RMSE	0.000374	0.000427	0.001012

Table 8: Forecasting prices results using MGBM.

	ABG.JO	**FSR.JO**	**SBK.JO**
MAPE (%)	5.495834	11.767967	8.309773

price paths over time. The errors in Table 8 show FSR.JO has the highest MAPE at 11.77%, meaning its prices are least accurately predicted. SBK.JO does better at 8.31% MAPE, while ABG.JO performs best with only 5.50% error. These results tell us that while the model can track general price movements, there are noticeable differences between predicted and actual prices, especially for FSR.JO

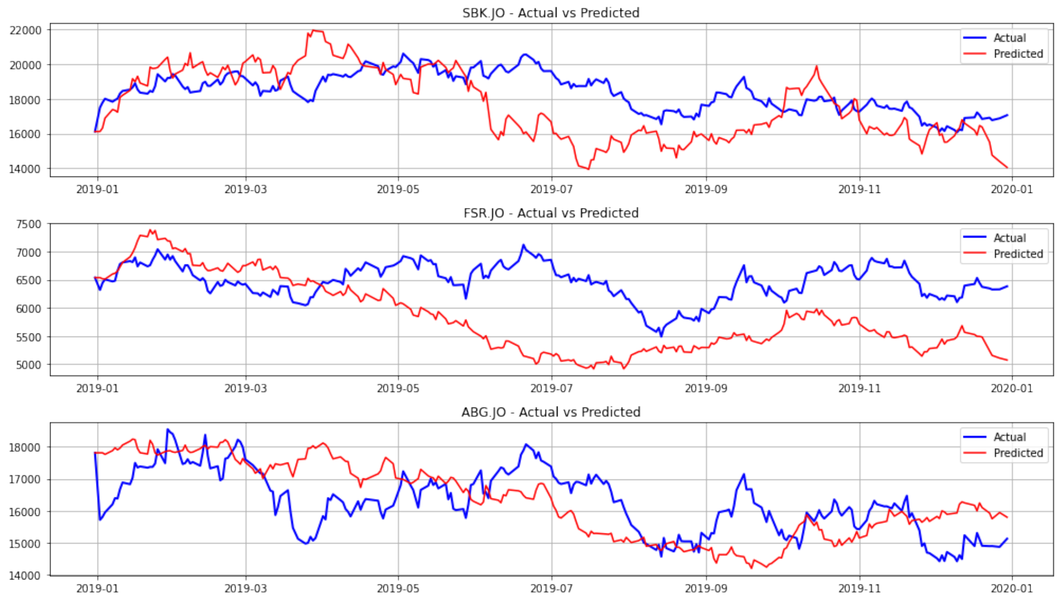


Fig. 5: Actual vs Predicted of ABG.JO, FSR.JO, SBK.JO.

Table 9: Return Forecasting Performance of DCC-GARCH Model.

	ABG.JO	**FSR.JO**	**SBK.JO**
MAE	0.016033	0.016339	0.017499
RMSE	0.000400	0.000420	0.000496

*4.3. **Performance of the DCC-GARCH model***

The return forecasting performance of the DCC-GARCH model was evaluated using MAE and RMSE. Figure 6 shows that the predicted and actual daily returns for all three stocks. Table 9 shows that SBK.JO has the highest prediction errors (MAE = 0.0175, RMSE = 0.0005), while ABG.JO and FSR.JO have similar results, with MAE around 0.0160 and RMSE close to 0.0004. These results suggest that the model captures return movements fairly well, although accuracy varies slightly between stocks.

For price predictions, Figure 7 shows the actual and forecasted price paths over time. The MAPE results in Table 10 show that SBK.JO has the most accurate price forecasts with an error of 3.97%, followed by FSR.JO at 5.08% and ABG.JO at 6.59%. This means that the DCC-GARCH model performs better at predicting SBK.JO's price movements compared to the other two stocks, even though its return forecasts had slightly higher errors.

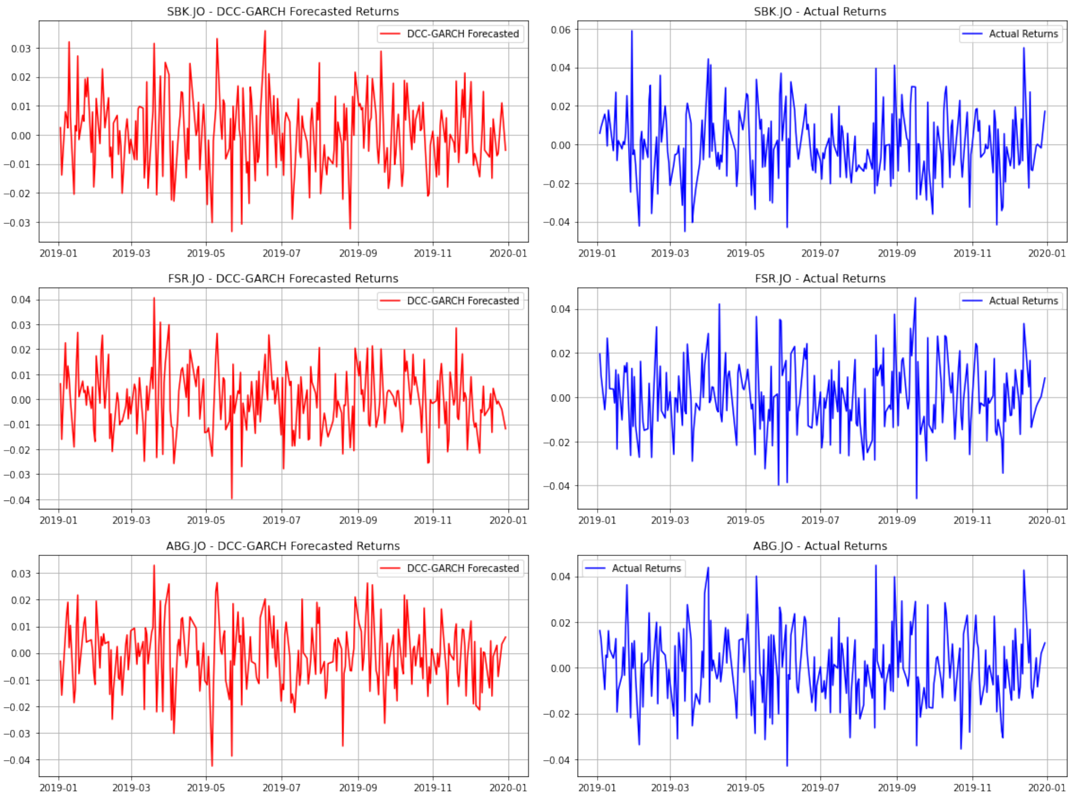


Fig. 6: Forecasted Returns from the DCC-GARCH Model.

Table 10: Price Forecasting Performance of the DCC-GARCH Model.

	ABG.JO	FSR.JO	SBK.JO
MAPE (%)	6.592007	5.079286	3.969346

4.4. *Performance Comparison of MGBM and DCC-GARCH Models*

A direct comparison between the MGBM model and the DCC-GARCH model highlights important differences in their forecasting behavior. When examining the performance on return predictions, both models show similar levels of accuracy for ABG.JO and FSR.JO, with only small differences in MAE and RMSE. For SBK.JO, however, the DCC-GARCH model produces lower errors, which suggests that it fits the more volatile structure of that asset's returns. When looking at price forecasts, the difference between the two models becomes clearer. The DCC-GARCH model produces lower MAPE values for two out of the three companies, meaning its predicted prices follow the actual market data more closely. This is especially noticeable for FSR.JO, where the error from the MGBM model is almost twice as

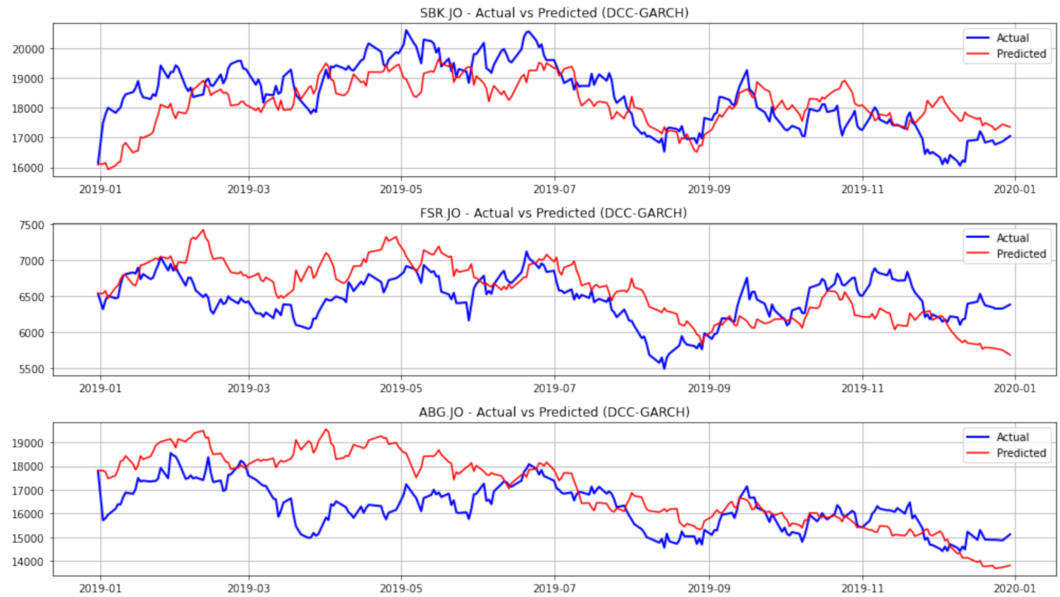


Fig. 7: Actual vs. Predicted Prices for ABG.JO, FSR.JO, and SBK.JO using the DCC-GARCH Model.

Table 11: Return Forecasting Comparison: GBM vs DCC-GARCH.

Metric	**ABG.JO**	**FSR.JO**	**SBK.JO**
GBM MAE	0.015933	0.016589	0.025864
GBM RMSE	0.000374	0.000427	0.001012
DCC-GARCH MAE	0.016033	0.016339	0.017499
DCC-GARCH RMSE	0.000400	0.000420	0.000496

high. For ABG.JO, MGBM performs slightly better, with a lower MAPE than DCC-GARCH. This relative underperformance of DCC-GARCH for ABG.JO may be due to its more stable return behavior, where the added complexity of the DCC-GARCH model does not translate into meaningful gains. These results show that the DCC-GARCH model is better at adjusting to changes in the market because it accounts for dynamic volatility and correlation, in contrast to the MGBM model, which uses fixed parameters. Overall, the comparison shows that while both models can capture the general behavior of returns and prices, the DCC-GARCH model tends to provide more precise forecasts, particularly in settings where market characteristics evolve over time.

Table 12: Price Forecasting MAPE: GBM vs DCC-GARCH.

	GBM MAPE (%)	DCC-GARCH MAPE (%)
ABG.JO	5.50	6.59
FSR.JO	11.77	5.08
SBK.JO	8.31	3.97

5. Conclusion and Future Works

The goal of this study was to examine and compare how well two common models in stochastic and financial time series analysis such as MGBM model and DCC-GARCH model, can predict stock returns and prices. We used historical daily closing prices from three large banks listed on the Johannesburg Stock Exchange (ABG.JO, FSR.JO, and SBK.JO) to test how accurately each model explains return patterns and forecasts future price changes over a set out-of-sample period. The GBM model assumes that constant drift and volatility and is a simple but widely used method for modeling asset prices. It was estimated using annualized log-return parameters from historical data and then used for future price trajectory simulation. On the other hand, the DCC-GARCH model allows volatility and correlations between assets to change over time. The parameters of this model is estimated in a two-stage procedure based on univariate GARCH(1,1) model and a subsequent dynamic conditional correlation. The DCC-GARCH model is made to better capture how financial markets change over time.

The results in our study showed that both models give fairly accurate forecasts for return series, with similar MAE and RMSE values for most of the stocks. However, the DCC-GARCH model performed much better than the MGBM for stocks with higher volatility, especially SBK.JO. Regarding the price forecasts, the DCC-GARCH model outperformed the MGBM model across all stocks, as shown by its consistently lower MAPE values. This shows that the DCC-GARCH model is good model at capturing changes in volatility and correlation over time features than the GBM model. However, the MGBM model is also useful because its limitations become clear when dealing with financial data that show patterns like volatility clustering and time-varying relationships between assets. The DCC-GARCH model can offer a more flexible structure that can adjust to changes in market behavior.

This flexibility helps to improve the accuracy of its forecasts. This study gives us to several directions for future research. One possible future would be to build a hybrid model that combines the MGBM and the DCC-GARCH model. Such a model could keep the simplicity of MGBM while adding the ability to account for changing volatility and correlation, which may improve its forecasting power. Another useful extension would be to test these models on larger portfolios, over longer forecast periods, and during different market conditions. A part from that, future research can also be the exploration of the traditional models combined with machine learning techniques to improve predictive accuracy.

In summary, this work highlights the simplicity and the flexibility in financial modeling. Both models examined in this study have their strengths, and combining them could lead to more effective ways of modeling financial data in practice.

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