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Comparative Analysis of Exchange Rate Models for Eastern African Countries

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Abstract. This paper presents a comprehensive study comparing three distinct models including Autoregressive Integrated Moving Average (*ARIMA*), Geometric Brownian Motion (*GBM*), and Artificial Neural Networks (*ANN*) for predicting Rwandan franc exchange rates in Eastern Africa countries. Utilizing historical exchange rate data from the National Bank of Rwanda (*BNR*), predictive models are constructed for each method. The analysis reveals that both the *ANN* and *ARIMA* outperform *GBM* in accurately approximating Rwandan franc exchange rates. These findings highlight the superior forecasting capabilities of *ARIMA* and *ANN* for Eastern African exchange rates, while providing insights into the performance of *GBM* in comparison.

Key words: time series; ARIMA; geometric Brownian motion; artificial neural networks; Eastern African Countries (EAC); exchange rate.
AMS 2010 Mathematics Subject Classification Objects : 60G05; 60H30; 65C60.

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Résumé. (Abstract in French) Cet article présente une étude complète comparant trois modèles distincts, notamment le modèle Autoregressive Integrated Moving Average (*ARIMA*), le modèle de Mouvement Brownien Géométrique (*GBM*) et les Réseaux de Neurones Artificiels (*ANN*), pour prédire les taux de change du franc rwandais dans les pays d'Afrique de l'Est. En utilisant les données historiques des taux de change de la Banque Nationale du Rwanda (*BNR*), des modèles prédictifs sont construits pour chaque méthode. L'analyse révèle que les modèles *ANN* et *ARIMA* surpassent le modèle *GBM* en termes de précision dans l'estimation des taux de change du franc rwandais. Ces résultats mettent en évidence les capacités de prévision supérieures des modèles *ARIMA* et *ANN* pour les taux de change en Afrique de l'Est, tout en fournissant des informations sur la performance du modèle *GBM* en comparaison.

Presentation of the authors.

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1. Introduction

Forecasting heavily relies on predictive modeling, a common mathematical technique used in many disciplines like business, social science, engineering, and finance. It is frequently referred to as predictive analytics in business. Predicting exchange rates is a key use of predictive modeling Clements and Lan (2010). The two primary categories of this contemporary method are statistical techniques and soft computing approaches Soltanali et al. (2021).

The *ARIMA* model is a versatile statistical tool that is frequently used in time series analysis and forecasting, including in the field of currency exchange rates Shumway et al. (2000). To make predictions from historical data, this model combines Autoregressive (*AR*), Integrated (I), and Moving Average (MA) components. However, it has difficulties with handling very seasonal currency data and relies on the assumption of linearity, which makes it sensitive to parameter selection Ariyo et al. (2014). However, the Geometric Brownian motion model finds its place in finance, particularly in currency exchange rate prediction, by assuming continuous and stochastic price changes, often characterized by log-normally distributed returns. Although it simplifies real-world market dynamics by assuming continuous volatility and drift, this can limit its capacity to reflect the entire range of financial intricacies Hull (2003). When navigating the complexities of currency exchange rate predictions, understanding the advantages and disadvantages of both methods is crucial.

On the other hand, due to their data-driven and self-adaptive character, Artificial Neural Networks (*ANN*s) stand out as outstanding and extremely powerful tools for predictive modeling. As universal function approximators, *ANN*s have the remarkable ability to recognize complicated correlations in data Goodfellow et al. (2016). It's fascinating to watch how the network generalizes; after learning from the available data, it can predict previously unobserved or future dataset components, even when the given data shows irregular patterns. *ANN*s are an effective and fascinating method for predicting jobs such as exchange rate prediction and other forecasting activities due to its inherent adaptability and capability for managing non-linear and non-smooth data Hua et al. (2010)

Hence, the purpose of this study is to compare the exchange rate model of the East African countries using, *ARIMA*, *ANN*s, and *GBM* models. Forecasts of exchange rates are important because they affect many economic decisions. Without knowing future exchange rates, it would be difficult for lenders to price loans, which would limit credit and investments and, in turn, have a negative impact on the economy. The paper is organized as follows: The literature review is presented in Section 2. Section 3 is devoted to the methodology used in this study. The results of our models are described in Section 4. Section 5 explores discussions of the results. Finally, Section 6 concludes this work.

2. Literature Review

According to the literature, many researchers have been working on the exchange rate models, including Plasmans et al. (1998), where the authors used macroeconomic models and leveraged the power of artificial neural networks (*ANN*s) in their quest to comprehend the non-linear nature of exchange rate interactions. They failed in their attempts to provide accurate monthly forecasts, though. Hu et al. (1998), on the other hand, adopted a different strategy and modeled the exchange rate with a non-linear dependence on its historical values, which led to their model outperforming straightforward linear models. It's noteworthy that they did not evaluate their model against a random walk. Hu et al. (1998)'s follow-up study using daily and weekly data demonstrated the superiority of *ANN*s over random walk models as a more reliable forecasting technique.

Similarly, due to the adaptable forecasting model, *ARIMA* has captured the interest of researchers in several fields. Kenny et al. (1998) used *ARIMA* to anticipate Irish inflation, while Mondal et al. (2014) used *ARIMA* to predict stock prices with success. Notable examples are Ngan (2013) who used *ARIMA* to test their stock market forecasting abilities. *ARIMA* has expanded its boundaries and been used by Guha and Bandyopadhyay (2016) to anticipate gold prices. Gupta and Dayal (2021) all show that *ARIMA* is an effective tool for forecasting foreign currency rates. *ARIMA* continues to enthrall academics with its adaptability and broad uses, providing insightful information and precise projections in a variety of fields.

Additionally, the Geometric Brownian Motion (*GBM*) has been extensively studied and used in the world of finance due to its ability to model the dynamics of financial assets over time. The *GBM* model has been used to analyze a variety of financial instruments, including stock prices, interest rates, commodity prices, and foreign exchange rates. The *GBM* model has been enhanced to include more complex components, like stochastic volatility and leaps, to better mimic the behavior of financial markets as it is explained by Mandelbrot (1963).

3. Methodology

This section delves into the mathematical underpinnings of the three different models included in our comparative analysis: *ARIMA*, *GBM*, and *ANN*s. Where as, *GBM* captures price dynamics, *ARIMA* takes advantage of time series characteristics, and *ANN*s make use of data-driven flexibility. We start by thoroughly examining the mathematical underpinnings of each model, which enables us to assess how well they can forecast exchange rates in Eastern African countires. *Python was used to execute these models.*

3.1. ARIMA Model

To derive the mathematical formula for the *ARIMA* model, we start with the basic principles of Autoregressive (*AR*) models, Moving Average (MA) models, and Differencing operations.

Autoregressive (*AR*) Component: The linear relationship between an observation and a specific number of lag observations is captured by the *AR* component. Let's take a look at an *AR*(p) model, where p is the autoregressive component's order. The general equation for an *AR*(p) model is Montgomery et al. (2015):

$$Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + E_t \tag{1}$$

where Y_t is the value of the time series at time t, c is a constant, $\phi_1, \phi_2, \ldots, \phi_p$ are the autoregressive parameters to be estimated, and E_t is the error term.

Differencing (I) Component: It is used to make the time series stationary by removing the trend and seasonality. Let's consider an I(d) model, where d is the order of differencing. The differencing operation involves computing the differences between consecutive observations. The differenced time series can be represented as Maindonald (2010):

$$Z_t = (1 - L)^d Y_t \tag{2}$$

where Z_t is the differenced time series, L is the lag operator, and Y_t is the original time series.

Moving Average (MA) Component: The MA component depicts the relationship between an observation and a residual error resulting from the application of a moving average model to lagged observations. Let's have a look at an MA(q) model, where q is the order of the moving average component. The general equation for an MA(q) model is Hyndman and Athanasopoulos (2018):

$$Y_t = \mu + E_t + \theta_1 * E_{t-1} + \theta_2 * E_{t-2} + \cdots + \theta_q * E_{t-q} \tag{3}$$

where μ is the mean of the time series, E_t is the error term at time t, and $\theta_1, \theta_2, \ldots, \theta_q$ are the moving average parameters to be estimated.

Combining the Autoregressive, Differencing, and Moving Average Components: The *ARIMA* model combines the autoregressive, differencing, and moving average components. The general equation for an *ARIMA*(p, d, q) model can be written as:

$$(1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p)(1 - L)^d Y_t = c + (1 + \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q) E_t \tag{4}$$

Expanding and rearranging the equation, we get:

$$(1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p)(1 - L)^d Y_t = c + E_t + \theta_1 E_{t-1} + \theta_2 E_{t-2} + \cdots + \theta_q E_{t-q} \tag{5}$$

which is the final form of the *ARIMA*(p, d, q) model equation as illustrated in Maindonald (2010). Therefore, from equation 5, the parameters that have been used were estimated as follows: The Autoregressive coefficient (ϕ_i) was calculated as

$$\hat{\phi}_i = \frac{\sum_{t=i}^{T} (Y_t - \hat{Y}_t)(Y_{t-2} - \hat{Y})}{\sum_{t=2}^{T} (Y_{t-2} - \hat{Y})^2}, \tag{6}$$

and the moving average coefficient (θ_i) as

$$\hat{\theta}_i = \frac{\sum_{t=2}^{T} (Y_t - \hat{Y}_t) E_{t-2}}{\sum_{t=2}^{T} (E_{t-2})^2}. \tag{7}$$

3.2. Geometric Brownian Motion

3.2.1. Description and Demonstration

In a stochastic process, the Geometric Brownian Motion (*GBM*) is used to simulate the long-term movement of exchange rates and other financial assets Sharpe (1964). It is presumptively true that the asset's price obeys a stochastic differential equation of the type

$$dS_t = \mu S_t dt + \sigma S_t dW_t, \tag{8}$$

where S_t is the exchange rate at time t, μ is drift, σ is the volatility and dW_t is the Wiener process (it captures the random and unpredictable fluctuations in a variable over time. In general it is responsible for introducing randomness into the *GBM* model).

Equation (8) can be written in a differential form by dividing both sides by S_t

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t. \tag{9}$$

Integrating both sides from 0 to t, we obtain

$$\ln\left(\frac{S_t}{S_0}\right) = \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t, \tag{10}$$

where S_0 is the initial stock price and W_t is the standard wiener process. Equation (10) can be written as

$$\begin{aligned} e^{\ln\left(\frac{S_t}{S_0}\right)} &= e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t} \\ \frac{S_t}{S_0} &= e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t} \\ S_t &= S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t}. \end{aligned} \tag{11}$$

This Equation (11) describes the behaviour of stock price, currency exchange rates and other financial disciplines Hull (2018).

3.2.2. Parameter estimation

Method 1: Maximum likelihood.

Equation (11) follows a log-normal distribution. The log-return, which is normally distributed with mean μ and standard deviation σ, can be expressed as

$$\begin{aligned} r_i &= \log\left(\frac{S(t_i)}{S(t_{i-1})}\right) \\ &= \left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\epsilon_i, \end{aligned} \tag{12}$$

where the observations $\{r_1, r_2, \ldots, r_n\}$ are log-returns calculated by considering current stock price to the previous one. Here Δt is the time interval between two consecutive observations, ϵ_i is a standard normal random variable and r_i is a log-return between t_{i-1} and t_i. Since the log-returns are normally distributed with mean and standard deviation, we can write $r_i \sim N(\mu, \sigma^2)$, with the probability density function (PDF) of the normal distribution given by:

$$f(r_i|\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(r_i-\mu)^2}{2\sigma^2}\right). \tag{13}$$

The joint probability density function of the log returns is given by

$$L(r_1, r_2, \ldots, r_n|\mu,\sigma) = \Pi_{i=1}^{n}\left(\frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(r_i-\mu)^2}{2\sigma^2}\right)\right). \tag{14}$$

Applying derivative on log-likelihood with respect to μ and σ, gives

$$\begin{aligned} \frac{d(\log L)}{d\mu} &= \frac{d}{d\mu}\sum_{i=2}^{n}\left(-\left(\frac{1}{2}\log(2\pi) + \frac{1}{2}\log(\sigma^2)\right) - \frac{(r_i-\mu)^2}{2\sigma^2}\right) \\ &= \frac{1}{2\sigma^2}\left(\sum_{i=2}^{n} -2r_t + 2\mu\right) \\ \frac{d(\log L)}{d\sigma} &= \sum_{i=2}^{n} -\frac{1}{\sigma} + \frac{(r_i-\hat{\mu})^2}{\sigma^3} \\ &= \sum_{i=2}^{n}\left(\frac{-\sigma^2 + (r_i-\hat{\mu})^2}{\sigma^3}\right). \end{aligned} \tag{15}$$

Setting these two equations to zero, gives

$$\begin{aligned} \hat{\mu} &= \frac{1}{n-1}\sum_{i=2}^{n} r_i \\ \hat{\sigma} &= \sqrt{\frac{1}{n-1}\sum_{i=2}^{n}(r_i-\hat{\mu})^2}. \end{aligned} \tag{16}$$

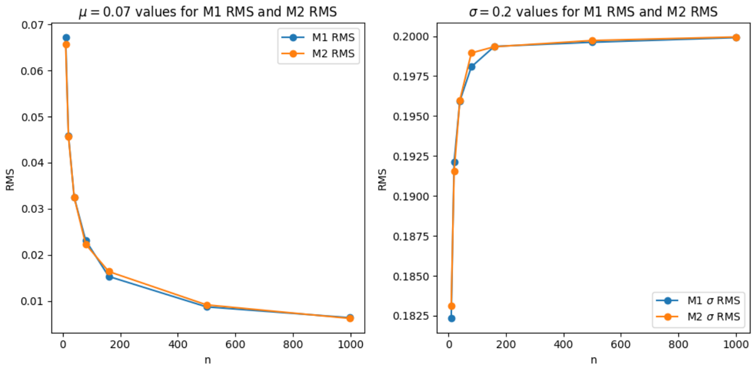

Fig. 1: Capability of Methods as n varies

Method 2: Method of Moment.

By assuming $S(t_i)$ is the stock price on the day, then the return from day (i) to the next day $(i+1)$ is given by

$$R_i = \frac{S(t_{i+1}) - S(t_i)}{S(t_i)} \tag{17}$$

then

$$\begin{aligned} \hat{\mu} &= \overline{R} = \frac{1}{n}\sum_{i=1}^{n} R_i \\ \hat{\sigma} &= \sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n}(R_i - \hat{\mu})^2}, \end{aligned} \tag{18}$$

These estimates provide us with a way to model and simulate stock prices using a *GBM*, and to make predictions about future price movements based on historical data.

To determine the most suitable method for this study, with fixed σ and μ. Various methods were considered. While the methods are almost the same, the method of Moments was selected for further analysis due to its lower computational complexity and precise results as illustrated in Figure 1.

3.3. Artificial Neural Network

A feedforward neural network is used in this section. This network is made up of interconnected layers of neurons that take inputs from sentiment in the news, past exchange rates, and economic data. Each neuron mixes its inputs using movable weights and biases, and then applies an activation function to generate an output through iterative computations. Using the backpropagation algorithm, the weights and biases of the network are tuned. The neural network catches complicated patterns and relationships by learning from historical data, providing precise forecasts of currency exchange rates in the future. Galeshchuk (2016). Let us see how this looks using a graphical representation:

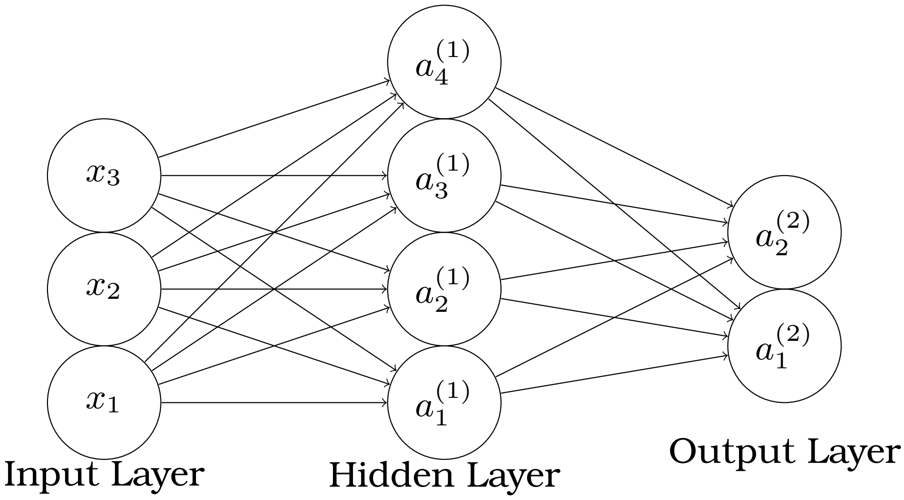

Fig. 2: Feedforward Neural Network

For a network with L layers, the output of each neuron can be calculated as follows Lawrence (1993):

Input layer:

$$x_i \quad \text{for } i = 1, 2, \dots, n \tag{19}$$

Hidden layer ($l = 1$ to $L - 1$):

$$\begin{aligned} z_j^{(l)} &= \sum_{i=1}^{n_{l-1}} w_{ij}^{(l)} a_i^{(l-1)} + b_j^l \\ a_j^{(l)} &= f\left(z_j^{(l)}\right) \end{aligned} \tag{20}$$

Output layer:

$$z_k^{(L)} = \sum_{j=1}^{n_{L-1}} w_{kj}^L a_j^{(L-1)} + b_k^{(L)}$$
$$a_k^{(L)} = f\left(z_k^{(L)}\right) \quad (21)$$

where:

- x_i represents the input to the neural networks
- n_0 represents the number of neurons in the input layer
- z_j^l represents the weighted sum of inputs to neuron j in layer l
- $w_{ij}^{(l)}$ represents the weight associated with the connection between neuron i in layer $l-1$ and neuron j in layer l
- $a_i^{(l-1)}$ represents the output of neuron i in layer $l-1$
- $b_j^{(l)}$ represents the bias associated with neuron j in layer l
- $f(.)$ represents the activation function applied element-wise to the weighted sum
- a_j^l represents the output of neuron j inn layer l after applying activation function
- $n_{(L-1)}$ represents the number of neurons in the previous layer before the output layer
- $z_k^{(l)}$ represents the weighted sum of inputs to the output neuron k
- $w_{(kj)}^L$ represents the weight associated with the connection between neuron j in the previous layer and output neuron k
- $b_k^{(l)}$ represents the bias associated with the output neuron k
- $a_k^{(L)}$ represents the final output of the neural network after applying the activation function

Rectified Linear Unit (ReLU) activation function was chosen as the activation function in this study because it is frequently employed in neural networks, particularly those that anticipate currency exchange rates. ReLU is renowned for its brevity, computing effectiveness, and capacity for vanishing gradient mitigation Yu et al. (2007). It has been demonstrated to function well in a variety of circumstances and has gained popularity as an activation function, which is modeled as

$$f(z) = max(0, z) \quad (22)$$

4. Comparative Analysis

4.1. Overview of financial data

The dataset has 2439 observations starting from January 4th, 2010, to October 16th, 2019, and was provided by the National Bank of Rwanda. It shows daily exchange rates for four currencies of East African Countries on Rwandan francs,

Table 1: Summary statistics for EAC members' currencies

	Ugsh	**Tzsh**	**Bif**	**Ksh**
Count	2439	2439	2439	2439
mean	0.2433	0.3869	0.4675	7.7183
std	0.0174	0.0240	0.0244	0.5717
min	0.1933	0.3080	0.4033	5.7594
25%	0.2313	0.3728	0.4442	7.3718
50%	0.2410	0.3862	0.4791	7.6670
75%	0.2550	0.4026	0.4871	8.0131
max	0.3007	0.4393	0.5069	8.9026

namely Burundi Francs (Bif/Rwf), Kenyan Shillings (Ksh/Rwf), Uganda Shillings (Ugsh/Rwf), and Tanzanian Shillings (Tsh/Rwf).
Considering time series data, we analyze exchange rate movements to derive crucial financial insights. Some rates exhibit positive exponential growth, suggesting increasing currency values, possibly as a result of economic growth. Conversely, declining rates imply values influenced by various economic and market circumstances.

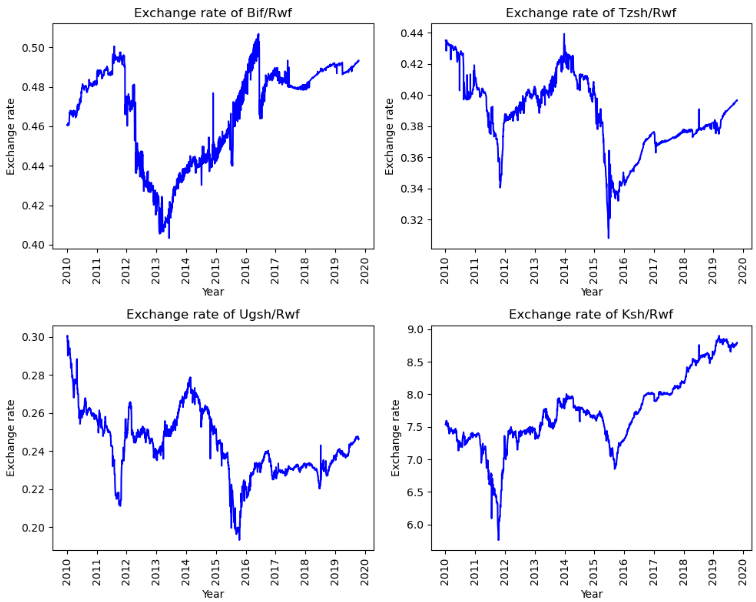

Fig. 3: Time series of EAC Exchange rate

Exciting exchange rate trends are revealed in this time series data illustrated in Figure 3, which have huge financial consequences. Positive exponential growth is shown by an upward trend in exchange rates, such as those for Bif and Ksh,

$\mu > \frac{\sigma^2}{2}$. This implies that the currencies values are rising over time, maybe due to good economic circumstances or rising demand for those currencies. Conversely, a declining trend in exchange rates, as seen for other rates, denotes a negative exponential growth rate $\mu < \frac{\sigma^2}{2}$. This suggests that the value of the currencies is dropping, which could potentially be caused by several factors, including slower economic growth, a drop in demand, or market forces that affect exchange rates. These findings align with the features of the (*GBM*) model, shedding light on how exchange rates fluctuate and providing valuable insights for financial analysis and decision-making.

4.2. Error checking

Mean Absolute Error (MAE) is a metric utilized to evaluate the average magnitude of errors between true values and the corresponding predicted values. MAE provides a straightforward way to evaluate the accuracy of a prediction or a simulation model.
MAE was calculated using this formula:

$$MAE(C_k) = \frac{1}{2439} \sum_{i=0}^{2438} |A(C_k)_i - S(C_k)_i|, \tag{23}$$

where, $C_k \in$ [Bif,Ksh,Tzsh,Ugsh], the actual values presented by

$$A(C_k) = [A(C_k)_0, A(C_k)_1, A(C_k)_2, \ldots,\ A(C_k)_{2438}],$$

and the estimated exchange rate using fitted model given by

$$S(C_k) = [S(C_k)_0, S(C_K)_1, \ldots, S(C_k)_{2438}].$$

MAE was useful because it gave equal weight to all errors without considering their direction (positive or negative). It provided a measure of the average magnitude of errors, allowing you to assess the overall accuracy or performance of a model.

Mean Forecast Error (MFE) was used to measure the accuracy of forecasting of the models and is given by:

$$MFE(C_k) = \frac{1}{2439} \sum_{i=0}^{2438} (A(C_k)_i - S(C_k)_i). \tag{24}$$

A lower MFE indicates a better accuracy, a negative MFE indicates underestimation while a positive MFE indicates an overestimation of the real data.

Mean Square Error (MSE) has been used also to measure the squared difference between the predicted and actual values since it gives an insight on how well the models' predictions align with the true values.
Mathematically, the MSE is computed as:

$$MSE(C_k) = \frac{1}{2439} \sum_{i=0}^{2438} (S(C_k)_i - A(C_k)_i)^2. \tag{25}$$

The better model performance is indicated by a lower MSE.

4.3. Forecasting Exchange Rate using GBM

This section explores the use of the Geometric Brownian Motion (*GBM*) model for predicting exchange rates. Estimated exchange rate will be generated based on the *GBM* formula and compared to the actual exchange rate to evaluate the model's accuracy.

The estimated exchange rates were generated based on the *GBM* formula using the daily exchange rate of four East African Community currencies: Kenyan shillings, Ugandan shillings, Tanzanian shillings, and Burundian francs versus the Rwandan stock price exchange rate. The estimated exchange rate were then compared to the actual exchange rate to evaluate the accuracy of the *GBM* model.

Figure 4 shows the plot of actual exchange rates and the estimated exchange rates using *GBM*, each done by using the drift and volatility estimated from real data of that exchange rate by using the method of moment that we selected in Section 3.2.

Table 2: EAC currencies (estimated vs actual) error in prediction

Error(EAC/rwf)	**Bif**	**Ugsh**	**Ksh**	**Tzsh**
MAE	0.0128	0.0085	0.17498	0.0115
MFE	0.000343	0.00003	-0.050741	-0.000158
MSE	0.00026	0.00012	0.05463	0.00023

From Figure 4 and the error detector presented in Table 2, which compares estimated and actual exchange rates based on the *GBM* model, we observe that the *GBM* model performs very well for the Ugandan shilling exchange rate, with an error of approximately 0.00003. For other exchange rates, such as the Burundian francs and Tanzanian shilling, the *GBM* also performs well, although not exactly like the Ugandan shilling. However, the Kenyan shilling is not performing as well compared to the others due to its higher error rate.

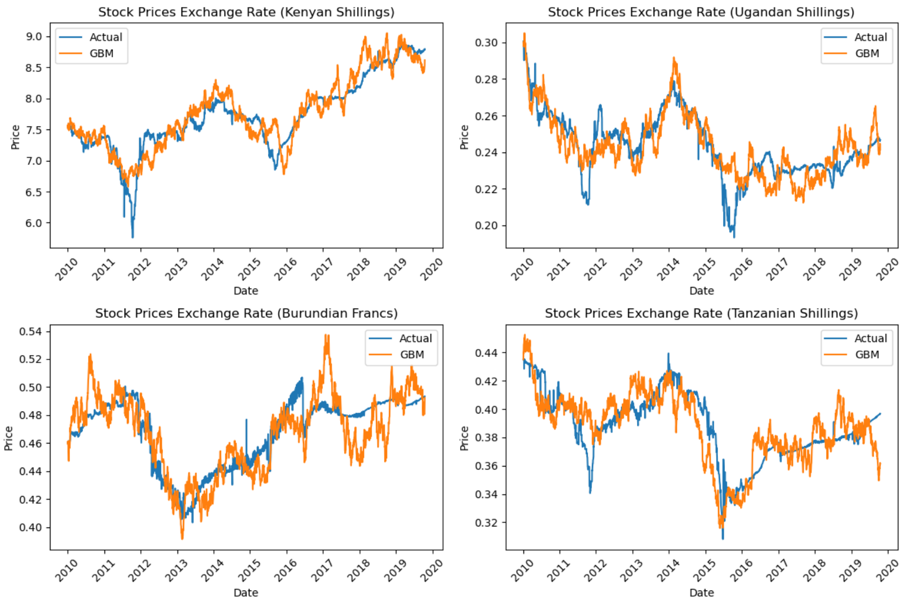

Fig. 4: Simulated vs Actual

4.4. Forecasting Exchange Rate using ARIMA

In this section we examine how the AutoRegressive Integrated Moving Average (*ARIMA*) model is used to forecast exchange rates. We estimated exchange rates using the *ARIMA* formula and compared them to the observed exchange rates to determine their performance.

The error detector illustrated in Table 3 offers helpful insights into the model's performance and its capacity to capture the dynamics of currency markets by providing a thorough summary of the prediction errors incurred while using the *ARIMA* model for exchange rate forecasting.

Table 3: EAC currencies (estimated vs actual) error in prediction

Error(EAC/rwf)	**Bif**	**Ugsh**	**Ksh**	**Tzsh**
MAE	0.0014	0.0009	0.01911	0.0010
MFE	0.00016	0.00003	-0.0266705	-0.00063
MSE	0.00027	0.00006	0.02789	0.00012

4.5. Forecasting Exchange Rate using ANN

Using the strength of artificial neural networks (*ANN*), the *ANN* framework employed in our analysis utilizes a neural network architecture with two hidden layers. This model generates estimated exchange rates, which are subsequently rigorously

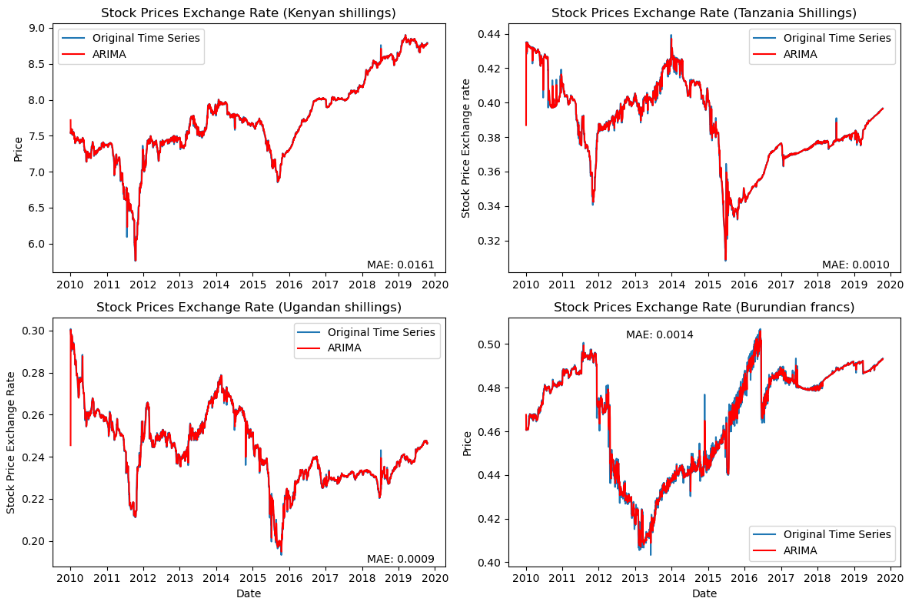

Fig. 5: *ARIMA* Result on data

evaluated in comparison to real exchange rate data. The outcomes of this analysis offer valuable insights into the effectiveness of the *ANN* model in capturing and forecasting dynamics within the currency exchange rate as illustrated in Figure 6.

Fig. 6: Estimated vs Actual

To evaluate how well the *ANN* model fitted the exchange rate dataset for East African countries, we employed MAE, MFE, and MSE as presented in Table 4. The model provides a succinct yet thorough description of the prediction errors observed during the forecasting of exchange rates using the *ANN* framework, offering a comprehensive snapshot of the model's accuracy and its ability to capture market dynamics.

Table 4: EAC currencies (simulated vs actual) error in prediction

Error(EAC/rwf)	**Bif**	**Ugsh**	**Ksh**	**Tzsh**
MAE	0.00224	0.00086	0.01911	0.00175
MFE	-0.00151	0.00006	-0.00160	-0.00111
MSE	0.00001	0.00000	0.00115	0.00001

5. Discussion

Table 5 provides an evaluation of various forecasting models' accuracy in relation to four East African exchange rate currencies to Rwandan francs. Three distinct performance metrics, MAE, MFE, and MSE, have been used to evaluate the performance of the model. The columns that correspond to each row indicate a particular predictive model and provide accurate outcomes for each currency.

Table 5: Accuracy of Models

Mean Absolute Error (MAE)				
Models	Ugandan (Ugsh/Rwf)	Tanzanian (Tzsh/Rwf)	Burundian (Bif/Rwf)	Kenyan (Ksh/Rwf)
ANN	99.914%	99.822%	99.776%	98.089%
GBM	99.15%	98.45%	98.72%	82.502%
ARIMA	99.91%	99.9%	99.86%	98.089%
Mean Forecast Error (MFE)				
ANN	99.994%	99.84%	99.9%	99.849%
GBM	99.997%	99.9842%	99.965%	94.9259%
ARIMA	99.997%	99.937%	99.984%	97.33%
Mean Square Error (MSE)				
ANN	99.9999%	99.999%	99.999%	99.885%
GBM	99.988%	99.977%	99.974%	94.537%
ARIMA	99.994%	99.988%	99.973%	97.211%

For the sake of readability and clarity of the results, Table 5 is color-coded. The color scheme acts as a visual signal of the levels of accuracy reached by these models, with different hues signifying various degrees of precision. In this color scheme,

1. The vibrant blue (99.90% and above) presents an excellent in high degree of accuracy
2. Invigorating green (98-98.9%) stands for very very good after vibrant blue
3. Pale yellow (98-98.9%) represent very good
4. Light orange (96-97.9%) stands for moderate accuracy
5. Dark gray (90-95.9%) indicates the lower degree of predictions
6. Light red (under 90%) lowest accuracy

However, by observing the results in Table (5), we have seen that

– MAE: Our MAE value of 97.91% indicates that the average absolute difference between the predicted exchange rates from our models and the actual exchange rates is roughly 97.91%. A model with a lower MAE value is often more accurate. A MAE of 97.91% in this instance indicates that our model's predictions are reasonably close to the actual rates, which is a positive result.
– MFE: for MFE result is 99.04%
– MSE: Our model exhibits relatively low variability in its prediction errors, with an MSE of 99.124%. A more stable model with reliable predictions has a lower MSE. This result shows a moderate level of prediction variance which is a good sign for the effectiveness of our models.

Practically speaking, these findings give us a thorough grasp of the advantages and shortcomings of our paradigm. The positive MFE indicates a tiny bias that might be corrected, while the low MAE signals that our models are generating accurate predictions. The moderate MSE indicates that the errors in our models are generally consistent, adding to its overall dependability.

5.1. Exchange rates

When we applied our study based on the data in the table to other East African currencies, we found that our predicted models' accuracy varied noticeably. Notably, the conversion rate for Ugandan Shillings was the most compatible with our models, with a remarkable accuracy percentage of 99.88%. With a superb accuracy level of 99.54%, the Tanzanian Shilling follows closely and performs really well. In contrast, the Burundian Shilling does well in predictions, attaining an impressive 99.5% accuracy. The Kenyan Shilling, on the other hand, trails its regional rivals by a small margin despite maintaining a relatively high accuracy rate of 95.88%.

The Ugandan Shilling stands out as the most properly forecasted currency, revealing important information about how well-suited our models are for predicting exchange rates in the East African currency landscape. The disparities in these currencies' accuracy highlight the significance of creating predictive models that are customized to each currency's unique characteristics in order to increase forecasting accuracy.

5.2. Model performance

Overall from Figure (4,5 and 6), it is clear that from looking at how well our models perform at forecasting exchange rates, we have attained remarkably high levels of accuracy. With a remarkable accuracy rate of 99.375%, the Artificial Neural Network (*ANN*) model has demonstrated excellent precision, according to our investigation. The *ANN* model clearly outperformed the alternative strategies, which highlights the effectiveness of using advanced machine learning techniques to estimate exchange rates.

In second place, we come upon the *ARIMA* model, which boasts a fantastic accuracy rate of 98.62%. The time series analysis-based *ARIMA* model is renowned for its capacity to identify temporal trends and patterns in exchange rates. Although not quite as precise as the *ANN* model, it nevertheless exhibits a high level of reliability when predicting exchange rates.

The Geometric Brownian motion model, which has a decent accuracy rate of 97.58%, comes in last. This strategy, which was motivated by the ideas of stochastic calculus, offers a fundamental comprehension of how asset prices change over time. Although it might not achieve the same degree of precision as the *ANN* and *ARIMA* models, its performance is nevertheless impressive and can be a useful tool in some forecasting circumstances.

6. Conclusion and Future Research Direction

Conclusion

Our analysis highlights a distinct hierarchy in the effectiveness of exchange rate prediction models. The Artificial Neural Network (*ANN*) model emerges as the top performer, showcasing its exceptional accuracy in estimations. Following closely is the *ARIMA* model, well-known for its precision in identifying temporal trends. The Geometric Brownian Motion model, securing the third rank, proves to be commendable and valuable for forecasting scenarios requiring a fundamental understanding of asset price fluctuations over time.

For decision-makers navigating the dynamic landscape of currency markets, these findings hold significant implications. The key takeaway is the crucial task of selecting the most suitable predictive model. While the *ANN* model stands out for its exceptional accuracy, the robust performance of the *ARIMA* model, particularly in capturing temporal trends, should not be underestimated. A nuanced understanding of each model's strengths becomes imperative for informed decision-making, facilitating alignment with the intricate currency landscape and enabling strategic choices for optimized accuracy.

Future Research Direction

Future studies could improve exchange rate forecasts by investigating hybrid models that combine the strengths of *ANN*, *ARIMA*, and Geometric Brownian Motion. To get a more complete picture, it would be beneficial to look into how outside variables such as international events, political influences and economic policies affect model performance. These models' ability to predict the dynamics of the currency market would be enhanced by continued improvement and adaptability to changing market conditions.

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